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SK Indices of Graph Operator $S(G)$ and $R(G)$ on few Nanostructures

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Abstract

In chemical graph theory the connectivity indices are applied to measure the chemical characteristics of compounds. In this paper, we compute the SK Indices of chemical structure graph operator subdivision graph $S(G)$ and semi total point graph $R(G)$ on certain important chemical structures like tetracenic nanotubes and tetracenic nanotori.

Keywords: Graph Operators, Topological Indices, Nanotubes

2010 MSC: 05C90, 05C35, 05C12

1. Introduction

Topological induces are numerical parameters of a graph which characterize its topology and are usually graph invariant. Topological indices are used in the development of quantitative structure-activity relationships (QSARs) in which the biological activity or other properties of molecules are correlated with their chemical structure.
The carbon nanotubes are types of nanostructure which are allotropes of carbon and having a cylindrical shapes. Carbon nanotubes a type of fullerence have potential in fields such as nano technology, electronics, optics, material science and architecture. Carbon nanotubes provides a certain potential for metal free catalysis of organic and inorganic reactions. [4, 5, 8]
In a chemical graph the number of vertices of G adjacent to a given vertex u is the *degree* of this vertex and will be denoted by d_u [3]. Results on Carbon nanocones related SK indices appear in [7].

2. Graph Operators and SK indices

This section focused on well known the definitions of subdivision of graphs, semi-total point graph and SK indices are evoked.

Definition 2.1. In [9, 10] the **subdivision graph** $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2 or equivalently by inserting an additional vertex into each edge of G.

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Definition 2.2. In [11, 14] the **semi-total point graph** $R(G)$ as defined as follows:
The operator $R(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it.

Definition 2.3. In [12, 13] V.S.Shigehalli and R.Kanabur introduced new degree based topological indices **(SK indices)** as follows.

$$SK(G) = \sum_{uv \in E(G)} \left[\frac{d_u + d_v}{2}\right]$$

$$SK_1(G) = \sum_{uv \in E(G)} \left[\frac{d_u . d_v}{2}\right]$$

$$SK_2(G) = \sum_{uv \in E(G)} \left[\frac{d_u + d_v}{2}\right]^2$$

In this paper, we are established the results on SK indices of subdivision graph and semi-total point graph of certain chemical structures like tetracenic nanotubes and tetracenic nanoturi.

2.1. SK indices of graph Operator on some Nanostructures

Linear [n] - Tetracene

The molecular graph of a linear[n]-Tetracenic is shown in Figure-1.

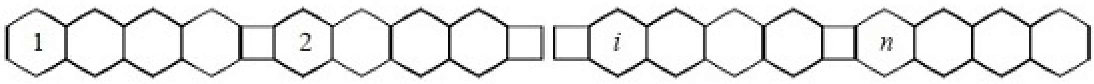

Figure 1 : The molecular graph of a linear[n]-Tetracene

Theorem 2.4. *Let T be a Linear[n]-Tetracene. Then*

$$1. SK[S(T)] = 107n - 14.$$
$$2. SK_1[S(T)] = 122n - 20.$$
$$3. SK_2[S(T)] = \tfrac{503}{2}n - 43.$$
$$4. SK[R(T)] = 290n - 44.$$
$$5. SK_1[R(T)] = 562n - 88.$$
$$6. SK_2[R(T)] = 1276n - 268.$$

Proof. Using Figure 1, apply algebraic method, we obtain for subdivision of T, i.e., $|V(S(T))| = 41n - 2$, and $|E(S(T))| = 46n - 4$. Also, the two edge partition sets of $S(T)$ as follows:

(2,2)	(2,3)
16n+8	30n-12

Let

$$SK(S(T)) = \sum_{uv \in E(S(T))} \left[\frac{d_u + d_v}{2}\right]$$

$= (16n + 8)\frac{2+2}{2} + (30n - 12)\frac{2+3}{2} = 107n - 14.$

Semi total point graph of T, $|V(R(T))| = 41n - 2$ and $|E(R(T))| = 69n - 6$. we obtain five edge partition sets of $R(T)$ as follows:

(2,4)	(2,6)	(4,4)	(4,6)	(6,6)
16n+8	30n-12	6	16n-4	7n-4

Consider,

$$SK(R(T)) = \sum_{uv \in E(R(T))} \left[\frac{d_u + d_v}{2} \right]$$

$=(16n+8)(\frac{2+4}{2}) + (30n-12)(\frac{2+6}{2}) + 6(\frac{4+4}{2}) + (16n-4)(\frac{4+6}{2})$
$+ (7n-4)(\frac{6+6}{2})$ $=290n-44$.
Similarly, we can prove the results (2),(3),(5) and (6).

□

3.2. Nanostructure F= F[p,q]

The molecular graph of 2-D lattice of F = F[p,q] with p=2 and q=4 is shown in Figure 2.

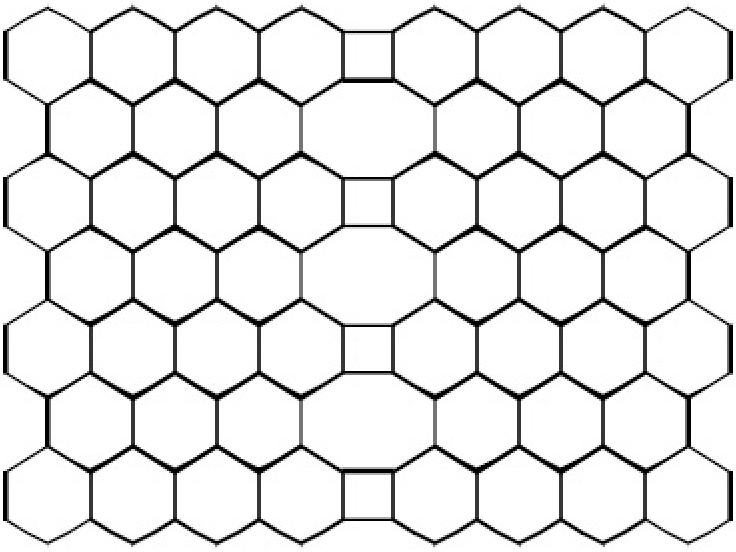

Figure 2 : the graph of 2-D lattice of F = F[p,q] with p=2 and q=4

Theorem 2.5. *Let $F = F[p,q]$ be a nanostructure. Then*

1.$SK[S(F)] = 135pq - 28p - 14q.$
2.$SK_1[S(F)] = 162pq - 40p - 20q.$
3.$SK_2[S(F)] = \frac{675}{2}pq - 86p - 43q.$
4.$SK[R(F)] = 378pq - 88p - 44q.$
5.$SK_1[R(F)] = 810pq - 248p - 152q + 40.$
6.$SK_2[R(F)] = 1836pq - 560p - 348q + 80.$

Proof. Let $F = F[p,q]$ be a nanostructure, by algebraic method, we obtain for subdivision of F. i.e., $|V(S(F))| = 27pq + 14p - 2q$, and $|E(S(F))| = 54pq - 8p - 4q$. We obtain two edge partition sets of $S(T)$ as follows:

(2,2)	(2,3)
16p+8q	54pq-24p-12q

Let

$$SK(S(F)) = \sum_{uv \in E(S(F))} \left[\frac{d_u + d_v}{2} \right]$$

$=(16p+8q)(\frac{2+2}{2}) + (54pq - 24p - 12q)(\frac{2+3}{2})$= $135pq - 28p - 14q$.

Semi total point graph of T, $|V(R(F))| = 27pq + 14p - 2q$ and $|E(R(F))| = 81pq - 12p - 6q$. we obtain five edge partition sets of $R(T)$ as follows:

(2,4)	(2,6)	(4,4)	(4,6)	(6,6)
16p+8q	54pq-24p-12q	2q+4	16p+4q-8	27pq-20p-8q+4

Consider,

$$SK(R(F)) = \sum_{uv \in E(R(F))} \left[\frac{d_u + d_v}{2}\right]$$

$=(16p + 8q)(\frac{2+4}{2}) + (54pq - 24p - 12q)(\frac{2+6}{2}) + (2q + 4)(\frac{4+4}{2})$
$+ (16p + 4q - 8)(\frac{4+6}{2}) + (27pq - 20p - 8q + 4)(\frac{6+6}{2})$ $=378pq - 88p - 44q$.
Similarly, we can prove the results (2),(3),(5) and (6).

□

3.3. Nanostructure G = G[p,q]

The molecular graph of 2-D lattice of $G = G[p, q]$ with p = 2 and q = 4 is shown in Figure 2.

Theorem 2.6. *Let $G = G[p, q]$ be a nanostructure. Then*

$$1. SK[S(G)] = 135pq - 28p.$$
$$2. SK_1[S(G)] = 162pq - 40p.$$
$$3. SK_2[S(G)] = \frac{675}{2}pq - 86p.$$
$$4. SK[R(G)] = 378pq - 88p.$$
$$5. SK_1[R(G)] = 810pq - 248p.$$
$$6. SK_2[R(G)] = 1836pq - 560p.$$

Proof. Let $G = G[p, q]$ be a nanostructure, by algebraic method, we obtain for subdivision of F.
$|V(S(G))| = 45pq - 4p$, and $|E(S(G))| = 54pq - 8p$. Also, obtain two edge partition sets of $S(T)$ as follows:

(2,2)	(2,3)
16p	54pq-24p

Let

$$SK(S(F)) = \sum_{uv \in E(S(F))} \left[\frac{d_u + d_v}{2}\right]$$

$=(16p)(\frac{2+2}{2}) + (54pq - 24p)(\frac{2+3}{2})$ $=135pq - 28p$.

Semi total point graph of T, $|V(R(G))| = 45pq - 4p$ and $|E(R(G))| = 81pq - 12p$. Also, obtain four edge partition sets of $R(T)$ as follows:

(2,4)	(2,6)	(4,6)	(6,6)
16p	54pq-24p	16p	27pq-20p

Consider,

$$SK(R(F)) = \sum_{uv \in E(R(F))} \left[\frac{d_u + d_v}{2}\right]$$

$=(16p)(\frac{2+4}{2}) + (54pq - 24p)(\frac{2+6}{2}) + (16p)(\frac{4+6}{2}) + (27pq - 20p)(\frac{6+6}{2})$
$=378pq - 88p$.
Similarly, we can prove the results (2),(3),(5) and (6).

□

3.4. Nanostructure K = K[p,q]

The molecular graph of 2-D lattice of $K = K[p, q]$ with p=2 and q=3 is shown in Figure 3.

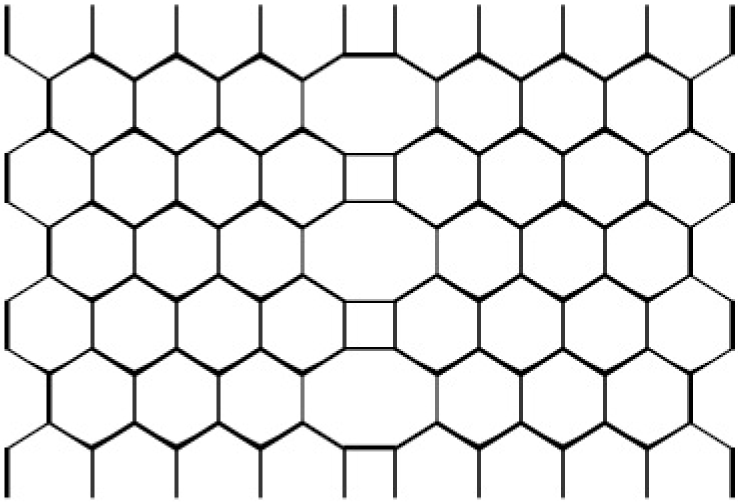

Figure 3 : the graph of 2-D lattice of $K = K[p, q]$ with p=2 and q=3

Theorem 2.7. *Let $K = K[p, q]$ be a nanostructure. Then*

$$1.SK[S(K)] = 135pq - 14q.$$
$$2.SK_1[S(K)] = 162pq - 20q.$$
$$3.SK_2[S(K)] = \frac{675}{2}pq - 43q.$$
$$4.SK[R(K)] = 378pq - 44q.$$
$$5.SK_1[R(K)] = 810pq - 120q.$$
$$6.SK_2[R(K)] = 1836pq - 276q.$$

Proof. Let $K = K[p, q]$ be a nanostructure, by algebraic method, we obtain for subdivision of F.
$|V(S(K))| = 45pq - 2q$, and $|E(S(K))| = 54pq - 4q$. Also, obtain two edge partition sets of $S(T)$ as follows:

(2,2)	(2,3)
8q	54pq-12q

Let

$$SK(S(K)) = \sum_{uv \in E(S(K))} \left[\frac{d_u + d_v}{2}\right]$$

$=(8q)(\frac{2+2}{2}) + (54pq - 12q)(\frac{2+3}{2})=135pq - 14q$
.

Semi total point graph of T, $|V(R(G))| = 45pq - 4p$ and $|E(R(G))| = 81pq - 12p$. Also, obtain five edge partition sets of $R(T)$ as follows:

(2,4)	(2,6)	(4,4)	(4,6)	(6,6)
8q	54pq-12q	2q	4q	27pq-8q

Consider,

$$SK(R(K)) = \sum_{uv \in E(R(K))} \left[\frac{d_u + d_v}{2}\right]$$

$=(8q)(\frac{2+4}{2}) + (54pq - 12q)(\frac{2+6}{2}) + (2q)(\frac{4+4}{2}) + (4q)(\frac{4+6}{2}) + (27pq - 8q)(\frac{6+6}{2})$
$=378pq - 44q$.
Similarly, we can prove the results (2),(3),(5) and (6).

□

3.5. Nanostructure L = L[p,q]

The molecular graph of 2-D lattice of $G = G[p, q]$ with p=2 and q=4 is shown in Figure 3.

Theorem 2.8. *Let $L = L[p, q]$ be a nanostructure. Then*

$$1.SK[S(L)] = 135pq.$$
$$2.SK_1[S(L)] = 162pq.$$
$$3.SK_2[S(L)] = \frac{675}{2}pq.$$
$$4.SK[R(L)] = 324pq.$$
$$5.SK_1[R(L)] = 594pq.$$
$$6.SK_2[R(L)] = 1350pq.$$

Proof. Let $L = L[p, q]$ be a nanostructure, by algebraic method, we obtain for subdivision of L.
$|V(S(L))| = 45pq$, and $|E(S(L))| = 54pq$. Also, obtain two edge partition sets of $S(L)$ as follows: (2,3)= 54pq
Let

$$SK(S(L)) = \sum_{uv \in E(S(L))} \left[\frac{d_u + d_v}{2}\right]$$

$=(54pq)(\frac{2+3}{2}) = 135pq$.

Semi total point graph of T, $|V(R(L))| = 45pq$ and $|E(R(L))| = 81pq$. Also, obtain four edge partition sets of $R(T)$ as follows:

(2,4)	(2,6)	(4,6)
27pq	27pq	27pq

$$SK(R(L)) = \sum_{uv \in E(R(L))} \left[\frac{d_u + d_v}{2}\right]$$

$=(27pq)(\frac{2+4}{2}) + (27pq)(\frac{2+6}{2}) + (27pq)(\frac{4+6}{2}) = 324pq$.
Similarly, we can prove the results (2),(3),(5) and (6). □

Acknowledgments

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