Montes Taurus J. Pure Appl. Math. 3 (2), 1–7, 2021

Montes Taurus Journal of Pure and Applied Mathematics
https://www.mtjpamjournal.com

MTJPAM
ISSN: 2687-4814

A Note on Dominator Chromatic Number of Graphs

Raghavachar Rangarajan [a], David Ashok Kalarkop [b]

[a]Department of Studies in Mathematics, University of Mysore, Manasagangothri, Mysuru – 570 006, India
[b]Department of Studies in Mathematics, University of Mysore, Manasagangothri, Mysuru – 570 006, India

Abstract

There are a lot of interesting results on dominator chromatic number $\chi_d(G)$ of a graph G, along with certain open problems in the recent literature. In this paper, an attempt is made to answer partially to some of the open problems. We study the effect on dominator chromatic number of graphs on removal (addition) of some edges in some classes of graphs. We also exhibit some non-isomorphic graphs having same dominator chromatic number and present some graphs such that $\chi_d(G) = \chi(G)$, $\chi_d(G) = \gamma(G)$ and $\chi_d(\mu(G)) = \chi_d(G) + 2$, where $\mu(G)$ is the Mycielskian of the graph G.

Keywords: Dominator coloring, Dominator chromatic number, Domination number

2010 MSC: 05C15, 05C69

1. Introduction

The graphs $G = (V, E)$ considered in the paper are simple graphs with vertex set V and edge set E. The order and size of the graph are $| V |= n$ and $| E |= m$ respectively. Neighbors of the vertex v of G are all those vertices which are adjacent to v in G and non-neighbors of the vertex v of G are all those vertices which are non-adjacent to v in G. The complement of the graph G denoted by $\overline{G}$ is the graph in which if two vertices are adjacent in G, then they are non-adjacent in $\overline{G}$ and vice-versa. Removal of vertex from a graph is a result obtained by removing the vertex and all the edges incident to the vertex from the graph. Removal of edge from a graph is result of removing the edges but not its end vertices from the graph. Two or more edges are said to be independent if they do not have common vertices. For more details on definitions and other terminologies we refer to [3, 5, 9].
Domination and coloring are the two important and interesting areas in graph theory. Domination has rich applications in computer science, communication networks and so on. Graph coloring has applications in scheduling, register allocations, pattern matching and so on (for more details refer [2]).
A subset S of V is said to be a dominating set of G if every vertex in $V - S$ is adjacent to some vertex in S. The minimum cardinality of dominating set of the graph G is said to be the domination number of the graph and is denoted by $\gamma(G)$. For more information on domination, one can refer [10, 11].
A proper coloring of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive same color. The minimum number of colors required for proper coloring of the graph G is said to be the chromatic number of G and is denoted by $\chi(G)$. The partition of the vertices of G into disjoint sets such that no two

†*Article ID:* MTJPAM-D-21-00012
Email addresses: rajra63@gmail.com (Raghavachar Rangarajan), david.ak123@gmail.com (David Ashok Kalarkop)
Received:8 January 2021, Accepted:4 March 2021, Published:31 March 2021
⋆*Corresponding Author:* David Ashok Kalarkop

vertices in the same set has same color is said to be the color partiton of $V(G)$ and the partitioned sets are said to be the color classes.
The dominator coloring of graph G is the proper coloring of G in such a way that every vertex of G dominates every vertex of atleast one color class. The minimum number of colors required for dominator coloring of the graph G is said to be the dominator chromatic number and is denoted by $\chi_d(G)$. The dominator coloring of the graph G using χ_d-colors is said to be the χ_d-coloring of G. For more results on dominator coloring of graphs we refer to [1, 7, 8].
The open neighborhood and closed neighborhood of the vertex v of G are $N(v) = \{u \in V(G) : uv \in E(G)\}$ and $N[v] = N(v) \cup \{v\}$ respectively. The vertex v of G with respect to the coloring C is said to be solitary if $\{v\} \in C$ and $N(v)$ does not contain any color class. Let $C = \{V_1, V_2, ..., V_k\}$ be the coloring of G. The color class V_i, $(1 \leq i \leq k)$ is said to be the spare color class with respect to C if every vertex $v \in V(G)$ dominates some color class V_j, $j \neq i$, of C in G. We need the following theorems to prove our main results.

Theorem 1.1. *[7]. Let G be a connected graph of order n. Then* $\chi_d(G) = n$ *if and only if G is complete graph* K_n.

Theorem 1.2. *[7]. For the cycle* C_n*, we have*

$$\chi_d(C_n) = \begin{cases} \lceil \frac{n}{3} \rceil, & \text{if } n = 4 \\ \lceil \frac{n}{3} \rceil + 1, & \text{if } n = 5 \\ \lceil \frac{n}{3} \rceil + 2, & \text{otherwise.} \end{cases}$$

Theorem 1.3. *[8]. For the path* P_n*, we have*

$$\chi_d(P_n) = \begin{cases} \lceil \frac{n}{3} \rceil + 1, & \text{if } n = 2, 3, 4, 5, 7 \\ \lceil \frac{n}{3} \rceil + 2, & \text{otherwise.} \end{cases}$$

Theorem 1.4. *[7]. The wheel* $W_{1,n}$ *has*

$$\chi_d(W_{1,n}) = \begin{cases} 3, & \text{if } n \text{ is even} \\ 4, & \text{if } n \text{ is odd.} \end{cases}$$

Theorem 1.5. *[1]. For any graph G,* $\chi_d(G) + 1 \leq \chi_d(\mu(G)) \leq \chi_d(G) + 2$. *Further if there exists a* χ_d*-coloring C of G in which every vertex v dominates a color class* V_i *with* $v \notin V_i$*, then* $\chi_d(\mu(G)) = \chi_d(G) + 1$.

Theorem 1.6. *[12]. Given a graph G,* $\chi_d(\mu(G)) = \chi_d(G) + 1$ *if and only if for some* χ_d-*coloring C of G:*
(i) each vertex v dominates some color class V_i *with* $v \notin V_i$*;*
(ii) a vertex v is a solitary vertex and C contains a spare color class V_i *which does not contain any vertex of* $N(v)$.

Mycielski [13] introduced a graph transformation called Mycielskian of a graph G denoted by $\mu(G)$, which is helpful for the construction of triangle free graphs and to increase the chromatic number of any graph by one. For more results on Mycielskian of a graph we refer to [4, 6].
In this paper, we study the effect on dominator chromatic number of graphs on removal (addition) of some edges in some classes of graphs. We also exhibit some non-isomorphic graphs having same dominator chromatic number and present some graphs such that $\chi_d(G) = \chi(G)$, $\chi_d(G) = \gamma(G)$ and $\chi_d(\mu(G)) = \chi_d(G) + 2$.

2. Effect of Dominator Chromatic Number of Some Graphs on Removal or Addition of Some Edges

In this section, we study the change in dominator chromatic number of graphs like complete graph, complete bipartite graph, wheel, cycle on removal or addition of some edges.

Theorem 2.1. *Let* K_n $(n \geq 3)$ *be the complete graph on n vertices. Removal of k edges* $(1 \leq k \leq n - 2)$ *incident to a vertex v of* K_n *will decrease the dominator chromatic number of* K_n *by* 1.

Proof. It is very trivial to observe that $\chi(K_n - \{e_1, e_2, e_3, \cdots, e_k\}) = \chi_d(K_n - \{e_1, e_2, e_3, \cdots, e_k\}) = n - 1$. □

Theorem 2.2. *Let k be the number of independent edges in* K_n*. Then removal of m* $(1 \leq m \leq k)$ *independent edges from* K_n *decreases the dominator chromatic number of* K_n *by m.*

Proof. The graph obtained by removal of m independent edges $\{e'_1, e'_2, e'_3, \cdots, e'_m\}$ from K_n will still have K_{n-m} has subgraph. So $n - m$ colors are required to color the subgraph K_{n-m}. Clearly the removal of each independent edge will give a pair of non-adjacent vertices out of which one vertex will be in K_{n-m}. Since each independent edge e'_i $(1 \leq i \leq m)$ has one vertex in subgraph K_{n-m} say v, then the other vertex can also be colored with color of v on removal of e'_i. This coloring will be the dominator coloring of $K_n - \{e'_1, e'_2, e'_3, \cdots, e'_m\}$. Therefore $n - m$ colors are the minimum number of colors required for proper coloring as well as dominator coloring of $K_n - \{e'_1, e'_2, e'_3, \cdots, e'_m\}$. Hence $\chi(K_n - \{e'_1, e'_2, e'_3, \cdots, e'_m\}) = \chi_d(K_n - \{e'_1, e'_2, e'_3, \cdots, e'_m\}) = n - m$. □

Corollary 2.3. *Let $(K_{n\times 2})$ be the cocktail party graph on $2n$ vertices. Then $\chi_d(K_{n\times 2}) = n$.*

Proof. Since $K_{n\times 2}$ is obtained from K_{2n} by removal of any n independent edges.
By Theorem 2.2, $\chi_d(K_{n\times 2}) = 2n - n = n$. □

Theorem 2.4. *Let $K_{m,n}$ $(m, n \geq 2)$ be the complete bipartite graph. Removal of k edges $\{e_1, e_2, e_3, \cdots, e_k\}$ $(1 \leq k \leq m$ or $1 \leq k \leq n$) incident to a vertex v of $K_{m,n}$ increases the dominator chromatic number of $K_{m,n}$ by 1.*

Proof. Two colors are sufficient for proper coloring of $K_{m,n} - \{e_1, e_2, e_3, \cdots, e_k\}$, but it is not the dominator coloring since the vertex v does not dominate any color class. Therefore vertex v has to be given a third color which will turn out to be the χ_d–coloring of $K_{m,n} - \{e_1, e_2, e_3, \cdots, e_k\}$. Hence $\chi_d(K_{m,n} - \{e_1, e_2, e_3, \cdots, e_k\}) = 3$. □

Theorem 2.5. *Dominator chromatic number of the complement of a $n-$ cycle $(n \geq 5)$ is $\lceil \frac{n}{2} \rceil$.*

Proof. Let $v_1 v_2 v_3 \cdots v_{n-1} v_n v_1$ be the $n-$ cycle C_n $(n \geq 5)$ whose complement is denoted by $\overline{C_n}$. Clearly v_1 and v_2 are non-adjacent in $\overline{C_n}$ and can receive the same color. If v_1 and v_2 have same color, then no other vertex of $\overline{C_n}$ can have the color of v_1 and v_2 since every vertex other than v_1 and v_2 is adjacent to either v_1 or v_2. Similarly v_3 and v_4 can have same color other than that of v_1 and v_2. Therefore the coloring $\{\{v_1, v_2\}, \{v_3, v_4\}, \ldots, \{v_{n-1}, v_n\}\}$ (if n is even) and $\{\{v_1, v_2\}, \{v_3, v_4\}, \ldots, \{v_{n-2}, v_{n-1}\}, \{v_n\}\}$ (if n is odd) will be the proper coloring as well as dominator coloring of $\overline{C_n}$. So $\chi_d(\overline{C_n}) \leq \lceil \frac{n}{2} \rceil$. It is easy to observe that maximum two vertices can be given same color in $\overline{C_n}$ since no three vertices are non-adjacent to each other. Hence $\chi_d(\overline{C_n}) \geq \lceil \frac{n}{2} \rceil$ and $\chi(\overline{C_n}) = \chi_d(\overline{C_n}) = \lceil \frac{n}{2} \rceil$. □

Theorem 2.6. *Let $W_{1,n}$ be the wheel graph on $n + 1$ vertices and $\{e_1, e_2, e_3, \cdots, e_n\}$ be the edges of the cycle C_n on $W_{1,n}$. Then*

$$\chi_d(W_{1,n} - \{e_1, e_2, e_3, \cdots, e_k\}) = \begin{cases} 3, & \text{if } (1 \leq k \leq n-1); \\ 2, & \text{if } k = n. \end{cases}$$

Proof. Since the removal of an edge e_i $(1 \leq i \leq n - 1)$ from $W_{1,n}$ will still have K_3 as a subgraph. So the minimum three colors are required for proper coloring of $(W_{1,n} - \{e_1, e_2, e_3, \cdots, e_k\})$ $(1 \leq k \leq n - 1)$. Color the central vertex by color 1 and the other two vertices of K_3 by color 2 and 3. The vertices of cycle C_n can be colored with either color 2 or 3. This coloring will be dominator coloring of $(W_{1,n} - \{e_1, e_2, e_3, \cdots, e_k\})$ and the minimum number of colors required for proper coloring as well as dominator coloring of $(W_{1,n} - \{e_1, e_2, e_3, \cdots, e_k\})$ $(1 \leq k \leq n - 1)$ is 3. On the removal of all the edges of C_n on $W_{1,n}$, graph will be $K_{1,n}$ whose dominator chromatic number is 2. □

Theorem 2.7. *Let $W_{1,n}$ be the wheel graph on $n + 1$ vertices and $v \in V(W_{1,n})$. For n even, joining v to all its non-neighbors increases the dominator chromatic number of $W_{1,n}$ by 1.*

Proof. We know that $\chi_d(W_{1,n}) = 3$, for n is even (by the Theorem 1.4).
The central vertex can be colored 1 and vertices of cycle C_n are consecutively colored with colors 2 and 3. Let $v \in V(W_{1,n})$ such that color of v is 2, then v is non-adjacent to vertices with color 2 or 3. Changing the color of non-neighbors of v to color 4 and then joining v to all its non-neighbors will result to dominator coloring of the new graph. So the minimum number of colors used for dominator coloring is 4. □

3. Construction of Some Graphs with $\chi_d(G_1) = \chi_d(G_2), \chi_d(G) = \gamma(G)$ and $\chi_d(G) = \gamma(G) + 1$

In this section we exhibit some non-isomorphic graphs with same dominator chromatic number, dominator chromatic number of a graph equal to the domination number of the graph or domination number plus one.

Theorem 3.1. *For $n \geq 2$, $\chi_d(K_{2n} - \{e'_1, e'_2, e'_3, \cdots, e'_n\}) = \chi_d(K_{2n-1} - \{e'_1, e'_2, e'_3, \cdots, e'_{n-1}\})$, where e'_i $(1 \leq i \leq n)$ are the independent edges.*

Proof. Clearly for $n \geq 2$, K_{2n} has n independent edges and K_{2n-1} has $n - 1$ independent edges. By the Theorem 2.2,
$\chi_d(K_{2n} - \{e'_1, e'_2, e'_3, \cdots, e'_n\}) = 2n - n = n$ and
$\chi_d(K_{2n-1} - \{e'_1, e'_2, e'_3, \cdots, e'_{n-1}\}) = 2n - 1 - (n - 1) = n$. □

Theorem 3.2. *Let $W_{1,n}$ be the wheel graph on $n + 1$ vertices and $v \in V(W_{1,n})$. For n odd, let H be the graph obtained by joining v to all its non-neighbors. Then $\chi_d(W_{1,n}) = \chi_d(H)$.*

Proof. We know that $\chi_d(W_{1,n}) = 4$, for odd n (by the Theorem 1.4). The central vertex can be colored 1 and vertices on the cycle C_n are consecutively colored with colors 2 and 3 and remaining one vertex with color 4. Let $v \in V(W_{1,n})$ be such that the color of v is 2. Then v is non-adjacent to vertices with color 2, 3 or 4. The non-neighbor vertex of v with color 2 can be changed by color 4 and then we can join v to all its non-neighbors which will result to the dominator coloring of H. The same process can be carried for the vertex with color 3. Now the vertex with color 4 is non-adjacent to the vertices of color 2 or 3 which will be dominator coloring even after joining vertex of color 4 to all its non-neighbors in $W_{1,n}$. So the minimum number of colors required for dominator coloring of H is 4. □

Theorem 3.3. *Let G be a graph with complete graph K_r as subgraph and $\{1, 2, 3, \cdots, r\}$ be the colors asssociated to the vertices $\{v_1, v_2, v_3, \cdots, v_r\}$ respectively of K_r in G. Let H be the graph obtained by adding a vertex v to G and joining it to every vertex of subgraph K_r in G. Let there exists a vertex v' in G such that color of $v' \notin \{1, 2, 3, \cdots, r\}$ and suppose v' dominates any color class $\{k\}(1 \leq k \leq r)$. Then,*
$\chi_d(G) = \chi_d(H)$.

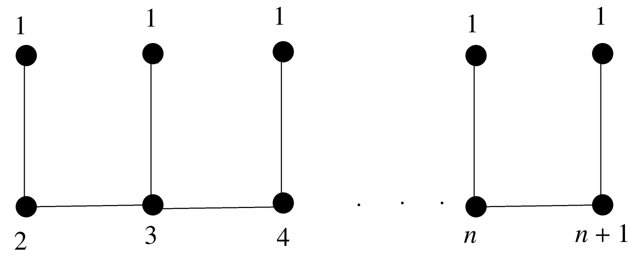
Proof. H is obtained from G by adding a vertex v and joining it to every vertex of K_r in G. Since the vertices of K_r are colored with colors $\{1, 2, 3, \cdots, r\}$, the vertex v cannot be colored with color $i(1 \leq i \leq r)$. Since there exists a vertex v' in G such that color of $v' \notin \{1, 2, 3, \cdots, r\}$ and v' dominates any color class $\{k\}(1 \leq k \leq r)$, the vertex v can be colored with the color of vertex v' which will lead to dominator coloring of graph H. □

Theorem 3.4. *Let G be the disjoint union of complete bipartite graphs. Then $\chi_d(G) = \gamma(G)$.*

Proof. Suppose $G = \cup G_i$ $(1 \leq i \leq k)$ be the disjoint union of k complete bipartite graphs G_i. Since G has k components and any two vertices from each component are required for domination. Therefore, $\gamma(G) = 2k$. Now for proper coloring of G, only two colors are sufficient but for the dominating coloring of G every component of G requires two new colors. Therefore $\chi_d(G) = 2k$ □

Theorem 3.5. *Let c_n be the $n-$centipede graph on $2n$ vertices. Then $\chi_d(c_n) = \gamma(c_n) + 1$.*

Proof. Since the $n-$centipede graph on $2n$ vertices is obtained by joining the bottoms of n copies of path P_2, so atleast one vertex from every path P_2 is required for domination.
Therefore $\gamma(c_n) = n$.
The minimum number of colors required for dominator coloring of c_n is $n + 1$. i.e the pendant vertices can be colored with color 1 and the vertices adjacent to pendant vertices with n distinct colors. The following figure shows the χ_d-coloring of c_n.



c_n

From the above figure it is clear that $\chi_d(c_n) = \gamma(c_n) + 1$. □

Theorem 3.6. *Let $W_{1,n}$ be the wheel graph on $n + 1$ vertices and $V(W_{1,n}) = \{v_0, v_1, v_2, \cdots, v_n\}$ with v_0 as the central vertex. For n even, let H be the graph obtained from $W_{1,n}$ by adding two vertices x and y, joining x to $\{v_{2i}\}$ $(1 \leq i \leq \frac{n}{2})$ and y to $\{v_{2i-1}\}$ $(1 \leq i \leq \frac{n}{2})$ and joining x and y. Then $\chi_d(W_{1,n}) = \chi_d(H)$ and $\chi_d(H) = \gamma(H) + 1$.*

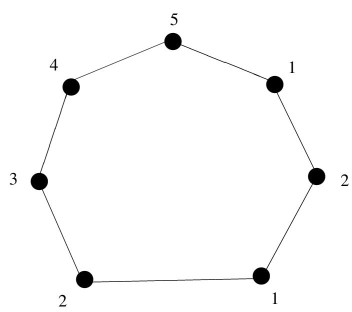
Proof. For n even, $\chi_d(W_{1,n}) = 3$ (by the Theorem 1.4). i.e color the vertex v_0 with color 1, $\{v_{2i}\}$ $(1 \leq i \leq \frac{n}{2})$ with color 2 and $\{v_{2i-1}\}$ $(1 \leq i \leq \frac{n}{2})$ with color 3. Now consider the graph H. Since x is adjacent to the vertices with color 2, x can be colored with color 3 and similarly y can be colored with color 2. Since x and y are adjacent in H, x dominates color class 2 and y dominates color class 3. Hence $\chi_d(H) = 3$.
No vertex of H is adjacent to all other vertices of H. So $\gamma(H) > 1$. Now $\{x, v_0\}$ will be the dominating set of H. Therefore $\gamma(H) = 2$ and hence $\chi_d(H) = \gamma(H) + 1$. □

4. On the Dominator Chromatic Number of Mycielskian of Some Graphs

In [1] it is proved that if there exists a χ_d-coloring of G such that every vertex v dominates a color class V_i with $v \notin V_i$, then $\chi_d(\mu(G)) = \chi_d(G) + 1$, where $\mu(G)$ is the Mycielskian of the graph G.
From results in previous sections, it is clear for graphs such as complete graph, complete bipartite graph, cocktail party graph, complement of $n-$ cycle, wheel that there exists a χ_d-coloring such that every vertex v dominates a color class V_i such that $v \notin V_i$. So dominator chromatic number Mycielskian of above mentioned graphs is dominator chromatic number of respective graph plus one.
Motivated by conclusion and scope in [1], we have proved some results to characterize the dominator chromatic number of Mycielskian of cycles.

Lemma 4.1. $\chi_d(\mu(C_7)) = \chi_d(C_7) + 1$.

Proof. we know that $\chi_d(C_7) = 5$ (by the Theorem 1.2). Consider the χ_d- coloring of C_7 has shown in the following figure.



Since every vertex of C_7 dominates a color class other than itself. By the Theorem 1.5 , $\chi_d(\mu(C_7)) = \chi_d(C_7)+1$. □

Theorem 4.2. *Let C_n $(n \geq 8)$ be the cycle on n vertices. Then for every χ_d−coloring of C_n, there exists a solitary vertex v.*

Proof. Let $v_1v_2v_3 \cdots v_{n-1}v_nv_1$ be the cycle on n vertices. Suppose there exists a χ_d−coloring of C_n $(n \geq 8)$ such that every vertex dominates a color class other than itself. Let S be the minimum dominating set of C_n. Clearly $| S |= \lceil \frac{n}{3} \rceil$. The dominator coloring of C_n such that every vertex dominates a color class other than itself can be done using altleast $2\lceil \frac{n}{3} \rceil$ colors. i.e, coloring the vertices of S with colors $1, 2, 3, \cdots, \lceil \frac{n}{3} \rceil$ and the vertices adjacent to vertex of color i $(1 \leq i \leq \lceil \frac{n}{3} \rceil)$ with color $\lceil \frac{n}{3} \rceil + i$. Then the vertex with color i dominates color class $\lceil \frac{n}{3} \rceil + i$ and the vertices with color $\lceil \frac{n}{3} \rceil + i$ dominates color class i. Therefore, the minimum number of colors required for dominator coloring of C_n such that every vertex dominates a color class other than itself is atleast $2\lceil \frac{n}{3} \rceil$ which is greater than the dominator chromatic number of C_n for $n \geq 8$ (by the Theorem 1.2). So the dominator coloring of C_n with atleast $2\lceil \frac{n}{3} \rceil$ colors is not χ_d−coloring of C_n which leads to a contradiction to our supposition. Therefore for every χ_d−coloring of C_n, there exists a solitary vertex v.

□

Theorem 4.3. *For every χ_d−coloring of C of C_n $(n \geq 8)$ and a solitary vertex v, there exists no spare color class which does not contain any vertex of $N(v)$.*

Proof. The solitary vertices with respect to the χ_d−coloring of C of C_n cannot be the spare color classes since they dominate themselves. Suppose the coloring C has the spare color class V_i which does not contain any vertex of $N(v)$, then the vertices of V_i should not be adjacent to any of the solitary vertex which will contradict the fact that $\chi_d(C_n) = \lceil \frac{n}{3} \rceil + 2$. Hence for every χ_d−coloring of C_n, there exists no spare color class which does not contain any vertex of $N(v)$.

□

In [1], S. Arumugam et al. demonstrated the dominator chromatic number of Mycielskian of cycles C_5, C_6 and have posed the following question,

For which cycles C_n, $\chi_d(\mu(C_n)) = \chi_d(C_n) + 2$?

The following result will characterize the dominator chromatic number of Mycielskian of cycles.

Theorem 4.4. $\chi_d(\mu(C_n)) = \chi_d(C_n) + 2$ *for* $n \geq 8$.

Proof. By the Theorem 4.2 and 4.3, it follows that for every χ_d−coloring of C_n $(n \geq 8)$, there exists a vertex v such that v dominates itself and for every solitary vertex v, there exists no spare color class which does not contain any vertex of $N(v)$ which is contrapositive to Theorem 1.6. Therefore $\chi_d(\mu(C_n)) = \chi_d(C_n) + 2$, for $n \geq 8$. □

5. Scope for Further Research.

The following are some interesting questions to carry out research in dominator coloring of graphs,

Problem 5.1. *Characterize graphs G such that there exists a solitary vertex for every χ_d−coloring of G.*

Problem 5.2. *Find graphs G such that* $\chi_d(\mu(G)) = \chi_d(G) + 2$.

Problem 5.3. *Find graphs G such that* $\chi_d(\mu(G)) = \chi_d(G) + 1$.

Problem 5.4. *Study the effect of change in dominator chromatic number on removal of cycles (or paths, complete graph etc) from the given graph.*

Acknowledgments

The authors are thankful to University Grants Commission (UGC), India for financial support under the grant UGC-SAP-DRS-II, NO.F.510/12/DRS-II/2018(SAP-I) dated: 9^{th} April 2018. The second author is thankful to UGC, New Delhi, for UGC-JRF, under which this work has been done. We thank Professor Chandrashekar Adiga for his valuable suggestions and discussions to improve the presentation of the paper.

References

[1] S. Arumugam, J. Bagga and K. R. Chandrasekar, *On dominator coloring in graphs*, Proc. Indian Acad. Sci. (Math. Sci.) **122 (4)**, 561-571, 2012.
[2] R. Balakrishnan and K. Ranganathan, *A Textbook of Graph Theory*, Springer, 2012.
[3] J. A. Bondy and U. S. R. Murty, *Graph Theory*, Springer, 2008.
[4] S. Brandt, *Triangle-free graphs whose independence number equals the degree*, Discrete Math. **310 (3)**, 662-669, 2010.
[5] G. Chartrand and L. Lesniak, *Graphs and Digraphs*, (4th edition) Chapman and Hall, CRC, 2011.
[6] X. Chen and H. Xing, *Domination parameters in Mycielski graphs*, Util. Math. **71**, 235-244, 2006.
[7] R. Gera, *On dominator coloring in graphs*, Graph Theory Notes N. Y. **52**, 25-30, 2007.
[8] R. Gera, C. Rasmussen and S. Horton, *Dominator colorings and safe clique partitions*, Congr. Numer. **181**, 19-32, 2006.
[9] F. Harary, *Graph Theory*, Narosa Publishing House, 2005.
[10] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, 1998.
[11] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Domination in Graphs-Advanced Topics*, Marcel Dekker, Inc., New York, 1998.
[12] A. Mohammed Abid and T. R. Ramesh Rao, *Dominator coloring of Mycielskian graphs*, Australas. J. Combin. **73(2)**, 274-279, 2019.
[13] J. Mycielski, *Sur le colouriage des graphes*, Colloq. Math. **3**, 161-162, 1955.