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Zagreb Indices of Square Snake Graphs

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Abstract

Several lattice structures that can be thought as graphs are useful in the study of large networks. In this work, we study 15 topological graph indices from the class of Zagreb indices of some interesting lattice structures called snake graphs. We use vertex and edge partitions of these graphs and calculate their indices by means of these partitions.

Keywords: Graph, Zagreb index, Vertex degrees, Graph index, Square snake graphs

2010 MSC: 05C07, 05C30, 05C38

1. Introduction

Snake graphs are studied in different contexts in mathematics and other sciences. They have finite or infinite one or two dimensional repetitions of some geometric shape. They are planar bipartite graphs. In [3, 4, 5, 6], the authors studied snake graphs in relation with cluster algebras. In [16], Shiffler constructed snake graphs consisting of square tiles and studied them in relation with perfect matchings and positive continued fractions which are used in estimating real numbers by some infinite sequences of rational numbers. Therefore the idea of continued fractions have been a popular and useful area of number theory. They are used in the solutions of some Diophantine equations. Bradshaw et al. continued the above work in [2] and established their relations with linear algebra by studying their characteristic polynomials in relation with the Chebycheff polynomials of the first and second type. In [10], snake graphs are studied in relation with strongly*-graphs.

In this work, we consider square snake graphs obtained by identifying a vertex of a square with a vertex of one or two neighbouring squares as in Fig. 1. It will be denoted by $C^1_{4,k}$.

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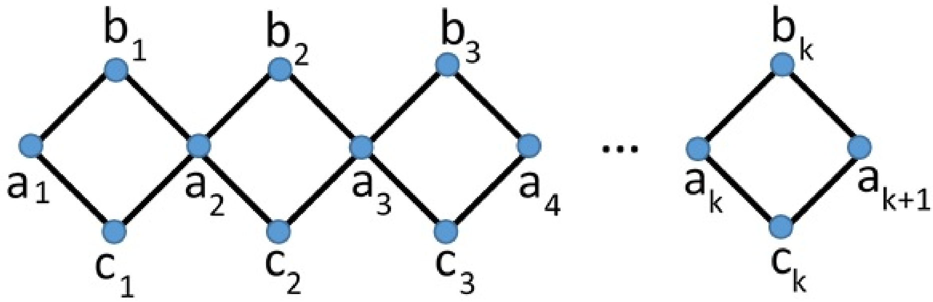

Figure 1 The square snake graph $C^1_{4,k}$

2. Some topological indices of square snake graphs

A topological index is a mathematical formula calculated by means of either vertex degrees, distances, or some graph parameters. In this section, we will determine some of the Zagreb type topological graph indices of the square snake graphs $C^1_{4,k}$. For several properties of Zagreb type graph indices, see [11, 12, 13, 14, 15, 17, 18, 19, 21]. The following Zagreb type indices are used in this work:

The first and second Zagreb indices are

$$M_1(G) = \sum_{v\in V(G)} d_G(v)^2 \text{ and } M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

They were defined nearly five decades ago by Gutman and Trinajstić. Some properties of these indices are studied in [7, 1, 8]. The forgotten index is defined as

$$F(G) = \sum_{v\in V(G)} d_G(v)^3$$

as a slight variant of the first Zagreb index. Its strange name comes from the fact that it did not get any attention for a long time after it was defined and used for the first time until Furtula and Gutman used it and named it as a forgotten index in [9].

As a slight variant of the first and second Zagreb indices, the generalized first and second Zagreb indices are defined as

$$M_1^\alpha(G) = \sum_{v\in V(G)} d_G(v)^\alpha$$

and

$$M_2^\alpha(G) = \sum_{uv\in E(G)} (d_G(u)d_G(v))^\alpha,$$

[7, 1]. Some properties of them are obtained in [20]. Similarly the generalized sum connectivity index is defined by

$$H_\alpha(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))^\alpha.$$

There are several variants of the classical graph indices in literature due to their applicative potential. Three of them, the redefined first, second and third Zagreb indices are defined as

$$ReZG_1(G) = \sum_{uv\in E(G)} \frac{d_G(u)+d_G(v)}{d_G(u)d_G(v)},$$

$$ReZG_2(G) = \sum_{uv\in E(G)} \frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}$$

and

$$ReZG_3(G) = \sum_{uv\in E(G)} d_G(u)d_G(v)(d_G(u) + d_G(v)).$$

The second Gourava index is defined by

$$M_{r,s}(G) = \sum_{uv\in E(G)}[d(u)^r.d(v)^s + d(u)^s.d(v)^r].$$

The reformulated first, second Zagreb indices and reformulated forgotten index are defined by

$$RM_1(G) = \sum_{uv\in E(G)}[d(uv)^2],$$

$$RM_2(G) = \sum_{e,e'\in E(G)}[d_G(e)\cdot d_G(e')]$$

and

$$RF(G) = \sum_{uv\in E(G)}[d(uv)^3].$$

Finally, the multiplicative first and second Zagreb indices are defined by

$$\prod_1(G) = \prod_{v\in V(G)} d(v)^2$$

and

$$\prod_2(G) = \prod_{uv\in E(G)} d(u)\cdot d(v).$$

Now we are ready to determine above topological graph indices of the square snake graph $C^1_{4,k}$.

Theorem 2.1. *The Zagreb type topological indices of the square snake graph $C^1_{4,k}$ are as follows:*

$$\begin{array}{ll}
M_1(C^1_{4,k}) & = 8(3k-1)\\
M_2(C^1_{4,k}) & = 16(2k-1)\\
F(C^1_{4,k}) & = 16(5k-3)\\
M_1^{\alpha}(C^1_{4,k}) & = (k+1).2^{\alpha+1} + (k-1)2^{2\alpha}\\
M_2^{\alpha}(C^1_{4,k}) & = 2^{2\alpha+2} + (k-1).2^{3\alpha+2}\\
H_{\alpha}(C^1_{4,k}) & = 4^{\alpha+1} + 4(k-1).6^{\alpha}\\
ReZG_1(C^1_{4,k}) & = 3k+1\\
ReZG_2(C^1_{4,k}) & = 4(4k-1)/3\\
ReZG_3(C^1_{4,k}) & = 64(3k-2)\\
M_{r,s}(C^1_{4,k}) & = 8\cdot 2^{r+s} + 4(k-1).2^{r+s}(2^r+2^s)\\
RM_1(C^1_{4,k}) & = 16(4k-3)\\
RM_2(C^1_{4,k}) & = 8(16k-15)\\
RF(C^1_{4,k}) & = 32(8k-7)\\
\prod_1(C^1_{4,k}) & = 2^{8k}\\
\prod_2(C^1_{4,k}) & = 2^{4(3k-1)}.
\end{array}$$

Proof. The square snake graph $C^1_{4,k}$ has $3k+1$ vertices and $4k$ edges. The vertex degrees are 2 and 4 and the vertex partition of $C^1_{4,k}$ is given in Table 1:

Table 1. Vertex partition of $C^1_{4,k}$

d_i	$\sharp\, d_i$
2	$2k+2$
4	$k-1$

Let us start with the first Zagreb index. We have

$$\begin{aligned} M_1(C^1_{4,k}) &= \textstyle\sum_{v\in V(G)} dv^2 \\ &= (2k+2)\cdot 2^2 + (k-1)\cdot 4^2 \\ &= 8k+8+16k-16 \\ &= 24k-8 \\ &= 8(3k-1) \end{aligned}$$

using Table 1. Also the edge partition of $C^1_{4,k}$ is shown in Table 2:

Table 2. Edge partition of $C^1_{4,k}$

(d_i, d_j)	$\sharp\,(d_i, d_j)$
$(2,2)$	4
$(2,4)$	$4(k-1)$

Now using Table 2, we calculate the second Zagreb index of the square snake graph $C^1_{4,k}$. From the edge partition of the square snake graph in Table 2, we have

$$\begin{aligned} M_2(C^1_{4,k}) &= \textstyle\sum_{uv\in E(G)} du\cdot dv \\ &= 4(2\cdot 2) + 4\cdot(k-1)\cdot(2\cdot 4) \\ &= 16+32(k-1) \\ &= 32k-16 \\ &= 16(2k-1) \end{aligned}$$

The forgotten Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} F(C^1_{4,k}) &= \textstyle\sum_{v\in V(G)} d(v)^3 \\ &= \textstyle\sum_{u,v\in E(G)}[d(u)^2+d(v)^2] \\ &= (2k+2)\cdot 2^3 + (k-1)\cdot 4^3 \\ &= 16k+16+64k-64 \\ &= 80k-48 \\ &= 16(5k-3). \end{aligned}$$

Also using the edge partition table, we can get an alternative proof of the same fact:

$$\begin{aligned} F(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[d(u)^2+d(v)^2] \\ &= 4\cdot(2^2+2^2) + 4(k-1)(2^2+4^2) \\ &= 32+80k-80 \\ &= 80k-48 \\ &= 16(5k-3). \end{aligned}$$

The general first Zagreb index of the square snake graph $C^1_{4,k}$ is found as

$$\begin{aligned} M_1^{\alpha}(C^1_{4,k}) &= \textstyle\sum_{v\in V(G)} d(v)^{\alpha} \\ &= (2k+2)\cdot 2^{\alpha} + (k-1)\cdot 4^{\alpha} \\ &= (k+1)\cdot 2^{\alpha+1} + (k-1)2^{2\alpha}. \end{aligned}$$

Also using the edge partition table, we have

$$\begin{aligned} F(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[d(u)^{\alpha-1}+d(v)^{\alpha-1}] \\ &= 4[2^{\alpha-1}+2^{\alpha-1}] + 4(k-1)\cdot[2^{\alpha-1}+4^{\alpha-1}] \\ &= 8\cdot 2^{\alpha-1} + 4(k-1)\cdot 2^{\alpha-1} + (k-1)4^{\alpha} \\ &= 2^{\alpha-1}(4k+4) + (k-1)\cdot 4^{\alpha} \\ &= (k+1)\cdot 2^{\alpha+1} + (k-1)\cdot 4^{\alpha}. \end{aligned}$$

The general second Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} M_2^{\alpha}(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[d(u)\cdot d(v)]^{\alpha} \\ &= 4(2\cdot 2)^{\alpha} + 4(k-1)(2\cdot 4)^{\alpha} \\ &= 4^{\alpha+1} + 4(k-1)8^{\alpha} \\ &= 2^{2\alpha+2} + (k-1)\cdot 2^{3\alpha+2}. \end{aligned}$$

The general sum connectivity index of the square snake graph $C^1_{4,k}$ is obtained by

$$\begin{aligned} H_{\alpha}(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[d(u)+d(v)]^{\alpha} \\ &= \textstyle\sum_{u,v\in E(G)}[d(uv)+2]^{\alpha} \\ &= 4(2+2)^{\alpha} + 4(k-1)(2+4)^{\alpha} \\ &= 4^{\alpha+1} + 4(k-1)\cdot 6^{\alpha}. \end{aligned}$$

The redefined first Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} ReZG_1(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[\frac{d(u)+d(v)}{d(u)\cdot d(v)}] \\ &= 4(\tfrac{4}{4}) + 4(k-1)(\tfrac{6}{8}) \\ &= 3k+1. \end{aligned}$$

Similarly the redefined second Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} ReZG_2(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)} \frac{d(u)\cdot d(v)}{d(u)+d(v)} \\ &= 4(\tfrac{4}{4}) + 4(k-1)(\tfrac{8}{6}) \\ &= (\tfrac{1}{3})\cdot(16k-4). \end{aligned}$$

Let us next calculate the first multiplicative Zagreb index. As

$$\begin{aligned} \textstyle\prod_1(C^1_{4,k}) &= \textstyle\prod_{v\in V(G)} d(v)^2 \\ &= 2^{2k+2}\cdot 4^2 k - 1 \\ &= 2^{8k}. \end{aligned}$$

The second multiplicative Zagreb index of the square snake graph $C^1_{4,k}$ is calculated by means of the edge partition table as

$$\begin{aligned} \textstyle\prod_2(C^1_{4,k}) &= \textstyle\prod_{u,v\in E(G)} d(u)\cdot d(v) \\ &= \textstyle\prod_{v\in V(G)} d(v)^{d(v)} \\ &= 2^{2(2k+2)}\cdot 4^{4(k-1)} \\ &= 2^{4(3k-1)}. \end{aligned}$$

The redefined third Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} ReZG_3(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)} d(u)d(v)\cdot[d(u)+d(v)] \\ &= (4\cdot 4)\cdot 4 + (6\cdot 8)\cdot 4(k-1) \\ &= 16(12k-8). \end{aligned}$$

For real numbers r and s, the second Gourava index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} M_{r,s}(C^1_{4,k}) &= \textstyle\sum_{u,v\in E(G)}[d(u)^r\cdot d(v)^s + d(u)^s\cdot d(v)^r] \\ &= 4[2^r\cdot 2^s + 2^s\cdot 2^r] + 4(k-1)[2^r\cdot 4^s + 2^s\cdot 4^r] \\ &= 4\cdot 2(2^{r+s}) + 4(k-1)[2^{r+2s} + 2^{s+2r}] \\ &= 8\cdot 2^{r+s} + 4(k-1)\cdot 2^{r+s}(2^s+2^r). \end{aligned}$$

The reformulated first Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} RM_1(C^1_{4,k}) &= \sum_{u,v\in E(G)} d(uv)^2 \\ &= \sum_{u,v\in E(G)}[d(u)+d(v)-2]^2 \\ &= 4[2+2-2]^2+4(k-1)[2+4-2]^2 \\ &= 64k-48. \end{aligned}$$

The reformulated second Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} RM_2(C^1_{4,k}) &= \sum_{e,e'\in E(G)}[d_G(e).d_G(e')] \\ &= (2\cdot 2)2+(2\cdot 4)4+(4\cdot 4)(6k-6) \\ &= 96n-56. \end{aligned}$$

And finally, the reformulated third (forgotten) Zagreb index of the square snake graph $C^1_{4,k}$ is

$$\begin{aligned} RF((C^1_{4,k})) &= \sum_{u,v\in E(G)} d(uv)^3 \\ &= 4\cdot(2+2-2)^3+4(k-1)(2+4-2)^3 \\ &= 256k-224. \end{aligned}$$

□

3. Summary and conclusions

In this work, square snake graphs are considered as special network structures and their Zagreb type topological indices are calculated.

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