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Four Unified Results for Reducibility of Srivastava's Triple Hypergeometric Series H_B

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Abstract

The aim of this paper is to provide four unified results of reducibility of the Srivastava's triple hypergeometric series H_B. The results are obtained with the help of two general results involving products of generalized hypergeometric series due to Rathie et al. A few known as well as unknown results follow as special cases of our main findings.

Keywords: Generalized hypergeometric functions, Summation formulas, Product formulas, Appell's functions, Kampé de Fériet function, Triple series

2010 MSC: 33C20, 33C60, 33A10, 33C77, 33C70, 33C65, 33C05

1. Introduction and Preliminaries

We begin by recalling the natural generalization of the Gauss's hypergeometric function ${}_2F_1$ namely the generalized hypergeometric function ${}_pF_q$ defined by [1, 4, 16, 24]

$$\begin{aligned}{}_pF_q\left[\begin{matrix}\alpha_1, \ldots, \alpha_p \\ \beta_1, \ldots, \beta_q\end{matrix}\,\middle|\, z\right] &= {}_pF_q(\alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_q; z), \\ &= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \cdot \frac{z^n}{n!},\end{aligned}$$

where $(a)_n$ is the well known Pochhammer symbol defined for $a \in \mathbb{C}$ by [24]

$$(a)_n = \begin{cases} 1, & (n = 0), \\ a(a+1)\ldots(a+n-1), & (n \in \mathbb{N}). \end{cases}$$

In terms of well-known Gamma function, it is written as

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} \quad (a \in \mathbb{C} \setminus \mathbb{Z}_0^-).$$

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For more details of ${}_2F_1$ and ${}_pF_q$ regarding their convergence conditions, various elementary properties, special cases and limiting cases we refer [1, 21].

It is not out of place to mention here that whenever Gauss's hypergeometric function ${}_2F_1$ or generalized hypergeometric function ${}_pF_q$ reduces to the Gamma functions, the results are very important from the application point of view. In this regard, several classical summation theorems for the series ${}_2F_1, {}_3F_2, {}_4F_3, {}_5F_4$ and others play an important role. Applications of these summation theorems are well-known. For this, we refer a very useful and interesting research paper by Bailey [3] in which several results involving products of generalized hypergeometric series are given. One of such results is given below:

$$ {}_0F_1\left[\begin{matrix} - \\ \rho \end{matrix}\,\middle|\, x\right] {}_0F_1\left[\begin{matrix} - \\ \rho \end{matrix}\,\middle|\, -x\right] = {}_0F_3\left[\begin{matrix} - \\ \rho, \frac{1}{2}\rho, \frac{1}{2}\rho+\frac{1}{2} \end{matrix}\,\middle|\, -\frac{x^2}{4}\right] \tag{1.1} $$

In 1973, Srivastava [23] extended the result (1.1) in the following form

$$ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n \frac{C_{m+n}\, x^{m+n}}{(\rho)_m\,(\rho)_n\, m!\, n!} = \sum_{n=0}^{\infty}(-1)^n \frac{C_{2n}}{(\rho)_n\,(\rho)_{2n}}\cdot\frac{x^{2n}}{n!} \tag{1.2} $$

where $\{C_n\}$ is a sequence of arbitrary complex numbers.

Another result due to Bailey [3] closely related to (1.1) is as follows:

$$ \begin{aligned} {}_0F_1\left[\begin{matrix} - \\ \rho \end{matrix}\,\middle|\, x\right] {}_0F_1\left[\begin{matrix} - \\ 2-\rho \end{matrix}\,\middle|\, -x\right] &= {}_0F_3\left[\begin{matrix} - \\ \frac{1}{2}, \frac{1}{2}\rho+\frac{1}{2}, \frac{3}{2}-\frac{1}{2}\rho \end{matrix}\,\middle|\, -\frac{x^2}{4}\right] \\ &\quad + \frac{2(1-\rho)x}{\rho(2-\rho)}\, {}_0F_3\left[\begin{matrix} - \\ \frac{3}{2}, \frac{1}{2}\rho+1, 2-\frac{1}{2}\rho \end{matrix}\,\middle|\, -\frac{x^2}{4}\right]. \end{aligned} \tag{1.3} $$

Bailey [3] and Srivastava [23] established these results with the help of the following classical Kummer's summation theorem [1] for the series ${}_2F_1$ viz.

$$ {}_2F_1\left[\begin{matrix} a, b \\ 1+a-b \end{matrix}\,\middle|\, -1\right] = \frac{\Gamma\left(1+\frac{1}{2}a\right)\,\Gamma(1+a-b)}{\Gamma(1+a)\,\Gamma\left(1+\frac{1}{2}a-b\right)} \tag{1.4} $$

Recently, good deal of progress has been done in the direction of generalizing and extending the classical summation theorems for the series ${}_2F_1$, ${}_3F_2$ and ${}_4F_3$. For this, we refer research papers by Lavoie et al. [11, 13, 12], Rakha and Rathie [17] and Kim et al. [9].

By employing the following two generalizations the Kummer's summation theorem (1.4) in the most general form for any $i \in \mathbb{N}_0$ obtained earlier by Rakha and Rathie [17] viz.

$$ {}_2F_1\left[\begin{matrix} a \quad b \\ 1+a-b+i \end{matrix}\,\middle|\, -1\right] = \frac{2^{-a}\Gamma\left(\frac{1}{2}\right)\Gamma(b-i)\Gamma(1+a-b+i)}{\Gamma(b)\Gamma\left(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}a-b+\frac{1}{2}i+1\right)} \sum_{r=0}^{i}\binom{i}{r}(-1)^r \frac{\Gamma\left(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2}r+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}a-\frac{1}{2}i+\frac{1}{2}r+\frac{1}{2}\right)} $$

and

$$ {}_2F_1\left[\begin{matrix} a \quad b \\ 1+a-b-i \end{matrix}\,\middle|\, -1\right] = \frac{2^{-a}\Gamma\left(\frac{1}{2}\right)\Gamma(1+a-b+i)}{\Gamma\left(\frac{1}{2}a-b+\frac{1}{2}i+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}a-b+\frac{1}{2}i+1\right)} \sum_{r=0}^{i}\binom{i}{r} \frac{\Gamma\left(\frac{1}{2}a-b-\frac{1}{2}i+\frac{1}{2}r+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}a-\frac{1}{2}i+\frac{1}{2}r+\frac{1}{2}\right)} $$

very recently Rathie et al. [19] generalized the results (1.1) and (1.3) due to Bailey in the most general form in the following forms for any $i \in \mathbb{N}_0$:

$$
\begin{aligned}
{}_0F_1\left[\begin{matrix} - \\ c \end{matrix} \,\middle|\, x\right] {}_0F_1\left[\begin{matrix} - \\ c+i \end{matrix} \,\middle|\, -x\right] &= k_1 \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}i+\frac{1}{2}r-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}r-\frac{1}{2}i\right)} \\
&\times {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i-\frac{1}{2}, \frac{1}{2}-\frac{1}{2}r+\frac{1}{2}i \\ \frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, c+\frac{1}{2}i-\frac{1}{2}, c+\frac{1}{2}i \end{matrix} \,\middle|\, -\frac{x^2}{4}\right] \\
&+ k_2 x \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}i+\frac{1}{2}r\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i\right)} \\
&\times {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i, 1-\frac{1}{2}r+\frac{1}{2}i \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i+1, c+\frac{1}{2}i, c+\frac{1}{2}i+\frac{1}{2} \end{matrix} \,\middle|\, -\frac{x^2}{4}\right], \qquad (1.5)
\end{aligned}
$$

where

$$k_1 = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(c+i)\Gamma(1-c-i)}{\Gamma(1-c)\Gamma\left(c+\frac{1}{2}i\right)\Gamma\left(c+\frac{1}{2}i-\frac{1}{2}\right)}$$

and

$$k_2 = \frac{-2\Gamma\left(\frac{1}{2}\right)\Gamma(c+i)\Gamma(-c-i)}{\Gamma(1-c)\Gamma\left(c+\frac{1}{2}i\right)\Gamma\left(c+\frac{1}{2}i+\frac{1}{2}\right)};$$

$$
\begin{aligned}
{}_0F_1\left[\begin{matrix} - \\ c \end{matrix} \,\middle|\, x\right] {}_0F_1\left[\begin{matrix} - \\ c-i \end{matrix} \,\middle|\, -x\right] &= k_3 \sum_{r=0}^{i} \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}r-\frac{1}{2}i-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i+\frac{1}{2}\right)} {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r-\frac{1}{2}i-\frac{1}{2}, \frac{1}{2}-\frac{1}{2}r+\frac{1}{2}i \\ \frac{1}{2}, \frac{1}{2}c, \frac{1}{2}c+\frac{1}{2}, c-\frac{1}{2}i, c-\frac{1}{2}i-\frac{1}{2} \end{matrix} ; -\frac{x^2}{4}\right] \\
&+ k_4 x \sum_{r=0}^{i} \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}r-\frac{1}{2}i\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i\right)} {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r-\frac{1}{2}i, 1-\frac{1}{2}r+\frac{1}{2}i \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1, c-\frac{1}{2}i, c-\frac{1}{2}i+\frac{1}{2} \end{matrix} \,\middle|\, -\frac{x^2}{4}\right] \qquad (1.6)
\end{aligned}
$$

where

$$k_3 = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma(c-i)}{\Gamma\left(c-\frac{1}{2}i\right)\Gamma\left(c-\frac{1}{2}i-\frac{1}{2}\right)}$$

and

$$k_4 = \frac{2\Gamma\left(\frac{1}{2}\right)\Gamma(c-i)}{c\,\Gamma\left(c-\frac{1}{2}i\right)\Gamma\left(c-\frac{1}{2}i+\frac{1}{2}\right)};$$

$$
\begin{aligned}
{}_0F_1\left[\begin{matrix} - \\ c \end{matrix} \,\middle|\, x\right] {}_0F_1\left[\begin{matrix} - \\ 2-c+i \end{matrix} \,\middle|\, -x\right] &= \frac{(-2)^i}{i!} \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(1-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r\right)}{\Gamma\left(1-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r\right)} \\
&\times {}_3F_6\left[\begin{matrix} 1, 1-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r, \frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r \\ \frac{1}{2}c, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}i+\frac{1}{2}, \frac{1}{2}i+1, 1-\frac{1}{2}c+\frac{1}{2}i, \frac{3}{2}-\frac{1}{2}c+\frac{1}{2}i \end{matrix} \,\middle|\, -\frac{x^2}{4}\right] \\
&+ \frac{4(-2)^i x}{(i+1)!c(2-c+i)} \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r+\frac{3}{2}\right)}{\Gamma\left(-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r+\frac{1}{2}\right)} \\
&\times {}_3F_6\left[\begin{matrix} 1, -\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r+\frac{3}{2}, \frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r+\frac{1}{2} \\ \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1, \frac{1}{2}i+1, \frac{1}{2}i+\frac{3}{2}, \frac{3}{2}-\frac{1}{2}c+\frac{1}{2}i, 2-\frac{1}{2}c+\frac{i}{2} \end{matrix} \,\middle|\, -\frac{x^2}{4}\right];
\end{aligned}
$$

$$
\begin{aligned}
{}_0F_1\left[\begin{matrix}-\\c\end{matrix}\,\middle|\,x\right]{}_0F_1\left[\begin{matrix}-\\2-c-i\end{matrix}\,\middle|\,-x\right] &= (-2)^i\sum_{r=0}^{i}\binom{i}{r}{}_2F_5\left[\begin{matrix}\frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r,1-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r\\ \frac{1}{2},\frac{1}{2}c,\frac{1}{2}c+\frac{1}{2},1-\frac{1}{2}c-\frac{1}{2}i,\frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i\end{matrix}\,\middle|\,-\frac{x^2}{4}\right]\\
&+\frac{2^{1-i}x}{c(2-c-i)}\sum_{r=0}^{i}\binom{i}{r}(1+r-i-c)\\
&\times\ {}_2F_5\left[\begin{matrix}\frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r,\frac{1}{2}+\frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r\\ \frac{3}{2},\frac{1}{2}c+\frac{1}{2},\frac{1}{2}c+1,\frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i,2-\frac{1}{2}c-\frac{1}{2}i\end{matrix}\,\middle|\,-\frac{x^2}{4}\right]. \qquad (1.7)
\end{aligned}
$$

Remark 1.1. The result (1.4) is also recorded in [15].

The great success of the theory of hypergeometric and generalized hypergeometric functions of a single variable have inspired the development of a corresponding theory into two and more variables. In 1880, Appell defined four functions popularly known in the literature as the Appell's functions F_1, F_2, F_3 and F_4 which are generalizations of the Gauss's hypergeometric functions. For a detailed account of the Appell's functions one may refer to [2, 26]. The confluent forms of the Appell's functions were studied by Humbert [5, 7]. However, here we would like to mention the definition of the ψ_2 as follows:

$$\psi_2(a;c,c';w,z)=\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{(a)_{k+m}w^kz^m}{(c)_k\,(c')_m\,k!m!} \qquad (1.8)$$

The following special case of (1.8) is worth mentioning [15] viz.

$$\psi_2(c;c,c;x,y)=\mathrm{e}^{x+y}{}_0F_1\left[\begin{matrix}-\\c\end{matrix}\,\middle|\,xy\right]. \qquad (1.9)$$

Later, the four Appell functions and their confluent forms were further generalized by Kampé de Fériet [8] who introduced a more general hypergeometric functions of two variables. In 1976, Srivastava and Panda [27] defined a more general double hypergeometric function than one defined by Kampé de Fériet. Just as the Gaussian hypergeometric function ${}_2F_1$ was generalized to ${}_pF_q$ by increasing the number of numerator and denominator parameters, the four Appell functions were unified and generalized to the Kampé de Fériet function. Later Lauricella [10] introduced fourteen complete hypergeometric function of three variables and of second order. Lauricella denoted his fourteen triple hypergeometric functions by the symbols

$$F_1,\ F_2,\ \ldots,\ F_{14}$$

of which F_1, F_2, F_5 and F_9 correspond, respectively, to the three variables Lauricella functions $F_A^{(3)}$, $F_B^{(3)}$, $F_C^{(3)}$ and $F_D^{(3)}$. For further details we refer [26]. The remaining ten functions of Lauricella's set apparently fell into oblivion [except that there is an isolated appearance of the triple hypergeometric function F_8 in a paper by Mayr [14] who came across this function while evaluating certain infinite integrals]. Saran [20] initiated a systematic study of these ten triple hypergeometric functions of Lauricella's set. In the course of further investigation of Lauricella's fourteen hypergeometric functions of three variables, Srivastava [22] noticed the existence of three additional complete triple hypergeometric functions of the second order. Srivastava denoted his functions by H_A, H_B and H_C which had neither been included in Lauricella's conjecture nor were previously mentioned in the literature. However here, in our present investigation we recall the series definition of H_B as follows [26]:

$$H_B(\alpha,\beta,\beta';\gamma_1,\gamma_2,\gamma_3;x,y,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\frac{(\alpha)_{m+p}(\beta)_{m+n}(\beta')_{n+p}x^my^nz^p}{(\gamma_1)_m(\gamma_2)_n(\gamma_3)_pm!n!p!},$$

provided $|x|<r$, $|y|<s$, $|z|<t$, with $r+s+t+2\sqrt{rst}=1$.

It is noted that H_B provides a generalization of the Appell's function F_2.

The following integral representation H_B was given by Srivastava [22]

$$H_B(a,b,b';c_1,c_2,c_3;x,y,z)=\frac{1}{\Gamma(a)\Gamma(b)}\int_0^\infty\!\!\int_0^\infty \mathrm{e}^{-s-t}s^{a-1}t^{b-1}\,{}_0F_1\left[\begin{matrix}-\\ c_1\end{matrix}\,\middle|\,xst\right]\psi_2(b';c_2,c_3;yt,zs)\,\mathrm{d}s\,\mathrm{d}t, \tag{1.10}$$

where ψ_2 is the confluent hypergeometric functions defined in (1.9).

In (1.10) if we take $c_1=c$, $c_3=c_2$, $b'=c_2$ and making use of the result (1.9) we get

$$H_B(a,b,c_2;c_1,c_2,c_2;x,y,z)=\frac{1}{\Gamma(a)\Gamma(b)}\int_0^\infty\!\!\int_0^\infty\left\{\mathrm{e}^{-(1-z)s-(1-y)t}s^{a-1}t^{b-1}{}_0F_1\left[\begin{matrix}-\\ c_1\end{matrix}\,\middle|\,xst\right]{}_0F_1\left[\begin{matrix}-\\ c_2\end{matrix}\,\middle|\,yzst\right]\right\}\mathrm{d}s\,\mathrm{d}t \tag{1.11}$$

Finally taking $c_1=c_2=c$, $x=-yz$ and making use of the result (1.1), we get the following interesting reducibility of H_B recoreded in [6, p. 136, Eq. (4.7.21)] viz.

$$(1-z)^a(1-y)^bH_B(a,b,c;c,c,c;-yz,y,z)={}_4F_3\left[\begin{matrix}\frac12a,\frac12a+\frac12,\frac12b,\frac12b+\frac12\\ c,\frac12c,\frac12c+\frac12\end{matrix}\,\middle|\,\frac{-4y^2z^2}{(1-y)^2(1-z)^2}\right] \tag{1.12}$$

The aim of this paper is to provide four unified result of reducibility of H_B (in the most general form), out of which two results are generalizations of the result (1.12) due to Srivastava. A few interesting special cases have also been given. These are achieved by the applications of the results (1.5) to (1.7).

The results established in this paper are simple, very interesting, easily established and may be potentially useful.

2. Main Results

The four unified results for reducibility of Srivastava's triple series H_B to be established in this paper are given in the following theorems.

Theorem 2.1. *Let $Y=2y/(1-y)$ and $Z=2z/(1-z)$. For $i\in\mathbb{N}_0$, the following two unified results hold true.*

$$\begin{aligned}1^\circ\quad (1-z)^a(1-y)^b\,H_B(a,b,c;c+i,c,c;-yz,y,z)&=k_1\sum_{r=0}^{i}(-1)^r\binom{i}{r}\frac{\Gamma\left(c+\frac12i+\frac12r-\frac12\right)}{\Gamma\left(\frac12+\frac12r-\frac12i\right)}\\&\quad\times{}_6F_5\left[\begin{matrix}c+\frac12r+\frac12i-\frac12,\frac12-\frac12r+\frac12i,\alpha\\ \frac12,\frac12c+\frac12i,\frac12c+\frac12i+\frac12,c+\frac12i-\frac12,c+\frac12i\end{matrix}\,\middle|\,-Y^2Z^2\right]\\&\quad+\frac14k_2\,ab\,YZ\sum_{r=0}^{i}(-1)^r\binom{i}{r}\frac{\Gamma\left(c+\frac12i+\frac12r\right)}{\Gamma\left(\frac12r-\frac12i\right)}\\&\quad\times{}_6F_5\left[\begin{matrix}c+\frac12r+\frac12i,1-\frac12r+\frac12i,\beta\\ \frac32,\frac12c+\frac12i+\frac12,\frac12c+\frac12i+1,c+\frac12i,c+\frac12i+\frac12\end{matrix}\,\middle|\,-Y^2Z^2\right],\end{aligned} \tag{2.1}$$

with $\{\alpha\}=\left\{\frac12a,\frac12a+\frac12,\frac12b,\frac12b+\frac12\right\}$, $\{\beta\}=\left\{\frac12a+\frac12,\frac12a+1,\frac12b+\frac12,\frac12b+1\right\}$, *and the constants k_1 and k_2 are the same as* *given in* (1.5).

$$2^\circ \quad (1-z)^a(1-y)^b H_B\,(a,b,c;c-i,c,c;-yz,y,z) = k_3 \sum_{r=0}^{i}\binom{i}{r}\frac{\Gamma\left(c+\frac{1}{2}r-\frac{1}{2}i-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i+\frac{1}{2}\right)}$$

$$\times\ {}_6F_5\left[\begin{matrix} c+\frac{1}{2}r-\frac{1}{2}i-\frac{1}{2}, \frac{1}{2}-\frac{1}{2}r+\frac{1}{2}i, \alpha \\ \frac{1}{2}, \frac{1}{2}c, \frac{1}{2}c+\frac{1}{2}, c-\frac{1}{2}i-\frac{1}{2}, c-\frac{1}{2}i \end{matrix}\ \middle|\ -Y^2Z^2\right]$$

$$+\ \frac{1}{4}k_4\, ab\, YZ \sum_{r=0}^{i}\binom{i}{r}\frac{\Gamma\left(c+\frac{1}{2}r-\frac{1}{2}i\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i\right)}$$

$$\times\ {}_6F_5\left[\begin{matrix} c+\frac{1}{2}r-\frac{1}{2}i, 1-\frac{1}{2}r+\frac{1}{2}i, \beta \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1, c-\frac{1}{2}i, c-\frac{1}{2}i+\frac{1}{2} \end{matrix}\ \middle|\ -Y^2Z^2\right], \qquad (2.2)$$

where $\{\alpha\} = \{\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\}$, $\{\beta\} = \{\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\}$ *and the constants* k_3 *and* k_4 *are the same as given in* (1.6).

Theorem 2.2. *Let* $Y = 2y/(1-y)$ *and* $Z = 2z/(1-z)$. *For* $i \in \mathbb{N}_0$, *the following two unified results hold true.*

$$1^\circ \quad (1-z)^a(1-y)^b H_B\,(a,b,c;2-c+i,c,c;-yz,y,z)$$

$$= \frac{(-2)^i}{i!}\sum_{r=0}^{i}(-1)^r\binom{i}{r}\frac{\Gamma\left(1-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r\right)}{\Gamma\left(1-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r\right)}$$

$$\times\ {}_7F_6\left[\begin{matrix} 1, 1-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r, \frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r, \alpha \\ \frac{1}{2}c, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}i+\frac{1}{2}, \frac{1}{2}i+1, 1-\frac{1}{2}c+\frac{1}{2}i, \frac{3}{2}-\frac{1}{2}c+\frac{1}{2}i \end{matrix}\ \middle|\ -Y^2Z^2\right]$$

$$+\ \frac{(-2)^i}{(i+1)!}\frac{abYZ}{c(2-c+i)}\sum_{r=0}^{i}(-1)^r\binom{i}{r}\frac{\Gamma\left(\frac{3}{2}+\frac{1}{2}i+\frac{1}{2}r-\frac{1}{2}c\right)}{\Gamma\left(\frac{1}{2}-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r\right)}$$

$$\times\ {}_7F_6\left[\begin{matrix} 1, \frac{3}{2}-\frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}r, \frac{1}{2}+\frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r, \beta \\ \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1, \frac{1}{2}i+1, \frac{1}{2}i+\frac{3}{2}, \frac{3}{2}-\frac{1}{2}c+\frac{1}{2}i, 2-\frac{1}{2}c+\frac{1}{2}i \end{matrix}\ \middle|\ -Y^2Z^2\right], \qquad (2.3)$$

where $\{\alpha\} = \left\{\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\right\}$ *and* $\{\beta\} = \left\{\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}\ , \frac{1}{2}b+1\right\}$.

$$2^\circ \quad (1-z)^a(1-y)^b H_B\,(a,b,c;2-c-i,c,c;-yz,y,z)$$

$$= 2^{-i}\sum_{r=0}^{i}\binom{i}{r}{}_6F_5\left[\begin{matrix} \frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r, 1-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r, \alpha, \\ \frac{1}{2}, \frac{1}{2}c, \frac{1}{2}c+\frac{1}{2}, 1-\frac{1}{2}c-\frac{1}{2}i, \frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i \end{matrix}\ \middle|\ -Y^2Z^2\right]$$

$$+\ 2^{-i-1}\frac{abYZ}{c(2-c-i)}\sum_{r=0}^{i}\binom{i}{r}(1+r-i-c)$$

$$\times\ {}_6F_5\left[\begin{matrix} \frac{1}{2}c+\frac{1}{2}i-\frac{1}{2}r+\frac{1}{2}, \frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i+\frac{1}{2}r, \beta \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1, \frac{3}{2}-\frac{1}{2}c-\frac{1}{2}i, 2-\frac{1}{2}c-\frac{1}{2}i \end{matrix}\ \middle|\ -Y^2Z^2\right], \qquad (2.4)$$

where $\{\alpha\} = \{\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\}$ *and* $\{\beta\} = \{\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\}$.

Proof. In order to establish the unified result (2.1) asserted in Theorem 2.1, we proceed as follows. In the integral representation (1.11) of H_B, if we set $c_2 = c$, $c_1 = c + i$, for $i = 0, 1, 2, \ldots$ and $x = -yz$, it takes the following form:

$$H_B(a,b,c;c+i,c,c;-yz,y,z) = \frac{1}{\Gamma(a)\Gamma(b)} \int_0^\infty \int_0^\infty \left\{ e^{-(1-z)s-(1-y)t} s^{a-1} t^{b-1} \, {}_0F_1\left[\begin{matrix} - \\ c \end{matrix} \middle| yzst\right] {}_0F_1\left[\begin{matrix} - \\ c+i \end{matrix} ; -yzst\right] \right\} \mathrm{d}s\,\mathrm{d}t.$$

Using the result (1.5), we have

$$H_B(a,b,c;c+i,c,c;-yz,y,z) = k_1 \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}i+\frac{1}{2}r-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}r-\frac{1}{2}i\right)} A + k_2 yz \sum_{r=0}^{i} (-1)^r \binom{i}{r} \frac{\Gamma\left(c+\frac{1}{2}i+\frac{1}{2}r\right)}{\Gamma\left(\frac{1}{2}r-\frac{1}{2}i\right)} B, \quad (2.5)$$

where

$$A = \frac{1}{\Gamma(a)\Gamma(b)} \int_0^\infty \int_0^\infty \left\{ e^{-(1-z)s-(1-y)t} s^{a-1} t^{b-1} \, {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i-\frac{1}{2},\ \frac{1}{2}-\frac{1}{2}r+\frac{1}{2}i \\ \frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, c+\frac{1}{2}i-\frac{1}{2}, c+\frac{1}{2}i \end{matrix} \middle| -\frac{y^2z^2s^2t^2}{4}\right] \right\} \mathrm{d}s\,\mathrm{d}t$$

and

$$B = \frac{1}{\Gamma(a)\Gamma(b)} \int_0^\infty \int_0^\infty \left\{ e^{-(1-z)s-(1-y)t} s^{a-1} t^{b-1} \, {}_2F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i,\ 1-\frac{1}{2}r+\frac{1}{2}i \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i+1, c+\frac{1}{2}i, c+\frac{1}{2}i+\frac{1}{2} \end{matrix} \middle| -\frac{y^2z^2s^2t^2}{4}\right] \right\} \mathrm{d}s\,\mathrm{d}t.$$

Evaluation of A. Expressing ${}_2F_5$ as a series, change the order of integration and summation, evaluating the integrals, after little simplification, using properties of Gamma function and then summing up the series, we finally have

$$A = (1-z)^{-a}(1-y)^{-b} \, {}_6F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i-\frac{1}{2}, \frac{1}{2}-\frac{1}{2}r+\frac{1}{2}i, \alpha \\ \frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, c+\frac{1}{2}i-\frac{1}{2}, c+\frac{1}{2}i \end{matrix} \middle| -Y^2Z^2\right], \quad (2.6)$$

where $\{\alpha\} = \left\{\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\right\}$, $Y = 2y/(1-y)$ and $Z = 2z/(1-z)$.

Evaluation of B. Proceeding on similar lines as in the case of evaluation of A, it is not difficult to see that

$$B = ab(1-z)^{-a-1}(1-y)^{-b-1} \, {}_6F_5\left[\begin{matrix} c+\frac{1}{2}r+\frac{1}{2}i, 1-\frac{1}{2}r+\frac{1}{2}i, \beta \\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}i+\frac{1}{2}, \frac{1}{2}c+\frac{1}{2}i+1, c+\frac{1}{2}i, c+\frac{1}{2}i+\frac{1}{2} \end{matrix} ; -Y^2Z^2\right] \quad (2.7)$$

where $\{\beta\} = \left\{\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\right\}$, and Y and Z are given before.

Finally, upon substituting the values of A and B from (2.6) and (2.7) in (2.5) and after some simplification, we easily arrive at the desired result (2.1). This completes the proof of our first unified result (2.1) asserted in Theorem 2.1.

In exactly the same manner, the other unified unified results (2.2), as well as ones in Theorem 2.2 can be established. □

3. Corollaries

In this section, we shall mention some of the very interesting known as well as unknown results of our main unified results. We always set $Y = 2y/(1-y)$ and $Z = 2z/(1-z)$.

Corollary 3.1. *In* (2.1) *or* (2.1)*, if we take* $i = 0$*, we get the known result* (1.12) *due to Srivastava [23].*

Corollary 3.2. *In* (2.1)*, if we take* $i = 1$*, we get*

$$(1-z)^a(1-y)^b H_B(a,b,c;c+1,c,c;-yz,y,z) = {}_4F_3\left[\begin{matrix} \frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2} \\ c, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}c+1 \end{matrix} \middle| -Y^2Z^2\right]$$

$$+ \frac{abYZ}{4c(c+1)} {}_4F_3\left[\begin{matrix} \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1 \\ c+1, \frac{1}{2}c+1, \frac{1}{2}c+\frac{3}{2} \end{matrix} \middle| -Y^2Z^2\right].$$

Corollary 3.3. *In* (2.1)*, if we take* $i = 2$*, we get*

$$(1-z)^a(1-y)^b H_B\,(a,b,c;c+2,c,c;-yz,y,z) = {}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ c+1,\frac{1}{2}c+1,\frac{1}{2}c+\frac{3}{2}\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$+\frac{2abYZ}{2c(c+2)}{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ c+1,\frac{1}{2}c+\frac{3}{2},\frac{1}{2}c+2\end{matrix}\,\middle|\,-Y^2Z^2\right].$$

Corollary 3.4. *In* (2.2)*, if we take* $i = 1$*, we get*

$$(1-z)^a(1-y)^b H_B\,(a,b,c;c-1,c,c;-yz,y,z) = {}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ c-1,\frac{1}{2}c,\frac{1}{2}c+\frac{1}{2}\end{matrix}\,;-Y^2Z^2\right]$$
$$-\frac{abYZ}{4c(c-1)}{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ c,\frac{1}{2}c+\frac{1}{2},\frac{1}{2}c+1\end{matrix}\,\middle|\,-Y^2Z^2\right].$$

Corollary 3.5. *In* (2.2)*, if we take* $i = 2$*, we get*

$$(1-z)^a(1-y)^b H_B\,(a,b,c;c-2,c,c;-yz,y,z) = {}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ c-1,\frac{1}{2}c,\frac{1}{2}c+\frac{1}{2}\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$-\frac{abYZ}{2c(c-2)}{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ c-1,\frac{1}{2}c+\frac{1}{2},\frac{1}{2}c+1\end{matrix}\,\middle|\,-Y^2Z^2\right].$$

In (2.3) or (2.4), if we take $i = 0$, with $Y = 2y/(1-y)$ and $Z = 2z/(1-z)$, we get

$$(1-z)^a(1-y)^b H_B\,(a,b,2-c;c,c,c;-yz,y,z) = {}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ \frac{1}{2},\frac{1}{2}c+\frac{1}{2},\frac{3}{2}-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$+\frac{(1-c)abYZ}{2c(2-c)}{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ \frac{3}{2},\frac{1}{2}c+\frac{1}{2},2-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right].$$

Corollary 3.6. *In* (2.3)*, if we take* $i = 1$*, we get*

$$(1-z)^a(1-y)^b H_B\,(a,b,c;3-c,c,c;-yz,y,z) = (2-c)\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ \frac{3}{2},\frac{1}{2}c+\frac{1}{2},\frac{3}{2}-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$-(1-c)\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}b,\frac{1}{2}b+\frac{1}{2}\\ \frac{3}{2},\frac{1}{2}c,2-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$+\frac{(2-c)ab\,YZ}{4(3-c)}\,{}_5F_4\left[\begin{matrix}1,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ 2,\frac{3}{2},\frac{1}{2}c+\frac{1}{2},\frac{5}{2}-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right]$$
$$+\frac{(1-c)ab\,YZ}{4c}\,{}_5F_4\left[\begin{matrix}1,\frac{1}{2}a+\frac{1}{2},\frac{1}{2}a+1,\frac{1}{2}b+\frac{1}{2},\frac{1}{2}b+1\\ 2,\frac{3}{2},\frac{1}{2}c+1,2-\frac{1}{2}c\end{matrix}\,\middle|\,-Y^2Z^2\right].$$

Corollary 3.7. *In* (2.3)*, if we take* $i = 2$*, we get*

$$(1-z)^a(1-y)^b H_B\left(a,b,c;4-c,c,c;-yz,y,z\right) = (2-c)^2\, {}_5F_4\left[\begin{matrix}1, \frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ 2, \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{5}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$-(1-c)(3-c)\,{}_5F_4\left[\begin{matrix}1, \frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ 2, \frac{3}{2}, \frac{1}{2}c, 2-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{(1-c)(2-c)(3-c)ab\,YZ}{6c(4-c)}\,{}_5F_4\left[\begin{matrix}1, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ 2, \frac{5}{2}, \frac{1}{2}c+1, 3-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{(2-c)abYZ}{6}\,{}_5F_4\left[\begin{matrix}1, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ 2, \frac{5}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{5}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right].$$

Corollary 3.8. *In* (2.4)*, if we take* $i = 1$*, we get*

$$(1-z)^a(1-y)^b H_B\left(a,b,c;1-c,c,c;-yz,y,z\right) = \frac{1}{2}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ \frac{1}{2}, \frac{1}{2}c, 1-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{1}{2}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ \frac{1}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{abYZ}{4(1-c)}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{3}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{abYZ}{4c}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ \frac{3}{2}, \frac{1}{2}c+1, 1-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right].$$

Corollary 3.9. *In* (2.4)*, if we take* $i = 2$*, we get*

$$(1-z)^a(1-y)^b H_B\left(a,b,c;-c,c,c;-yz,y,z\right) = \frac{1}{2}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ \frac{1}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{1}{2}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a, \frac{1}{2}a+\frac{1}{2}, \frac{1}{2}b, \frac{1}{2}b+\frac{1}{2}\\ \frac{1}{2}, \frac{1}{2}c, -\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{abYZ}{4c}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ \frac{3}{2}, \frac{1}{2}c+1, 1-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right]$$
$$+\frac{abYZ}{4c}\,{}_4F_3\left[\begin{matrix}\frac{1}{2}a+\frac{1}{2}, \frac{1}{2}a+1, \frac{1}{2}b+\frac{1}{2}, \frac{1}{2}b+1\\ \frac{3}{2}, \frac{1}{2}c+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}c\end{matrix}\middle| -Y^2Z^2\right].$$

Similarly, other results can be obtained.

Remark 3.10. For generalizations of the result (1.2), see Rathie [18].

4. Concluding Remarks

In this paper, we have established four unified results of reducibility of the Srivastava's triple hypergeometric series H_B. The results are obtained with the help of two general results involving products of generalized hypergeometric series due to Rathie et al. The results established in this paper are simple, very interesting, easily established and may be potentially useful.

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