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Mathematical Modelling of COVID-19 Using Approximated Hyper-Singular Integral Based Chebyshev Polynomials of the Second Kind

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Abstract

There are different investigations unfilled the population energetic of COVID-19. In this work, we formulate an approximated hypersingular integral based Chebyshev polynomials of second kind to simulate COVID-19 growth. The planned scheme indicates an association consequences of integral equation model by employing live data from Malaysia for three different months. MATLAB code is developed to obtain the numerical results for the presented problem. Moreover, the error function is applied to determine the compact interval of the infected number. We could establish agreement action on the displays where the numerical results assert the theoretical concept.

Keywords: Singular integrals, Hypersingular integral, Chebyshev polynomial, COVID-19

2010 MSC: 11T06, 11T22

1. Introduction

In numerical analysis, the numerical integration field plays a significant role in a broad family of algorithms for calculating the numerical value of a definite integral, and by extension, the term is also sometimes used to describe the numerical solution of differential equations and various other scientific aspects [6]. The basic problem considered by numerical integration is to compute an approximate solution to a definite integral

$$\int_a^b \xi(t)dt \tag{1.1}$$

If $\xi(t)$ is a smooth well-behaved function, integrated over a small number of dimensions and the limits of integration are bounded, then there are many methods of approximating the integral with arbitrary precision. The integral in (1.1) is also called Fredholm integral type because of the integral limits. It is vital to mention that there are several reasons

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for carrying out numerical integration. The integrand $\xi(t)$ may be known only at certain points, such as obtained by sampling. Some embedded systems and other computer applications may need numerical integration for this reason. The basic method involved in approximating problems presented by (1.1) is called numerical quadrature that uses the following sum [1]

$$\sum_{i=0}^{n} \alpha_i \xi(t_i).$$

The methods of quadrature are based on polynomial approximation, which takes the same values as the function at a certain number of points in the domain of the function. In other utterance, we consider the problem of determining a polynomial $P(t)$ of degree at most n that passes through various points like (x_0, y_0) and (x_1, y_1), is the same as approximating a function ξ for which $\xi(x_0) = y_0$ and $\xi(x_1) = y_1$ by means of polynomial approximation, the values of ξ at the given points take the form

$$\xi(t_i) = P_n(t_i)\ ; \qquad i = 0, 1, 2, \cdots, n$$

then $P(t)$ is called the interpolating polynomial as it exactly create the same data and it takes the form [12]

$$P(t) \approx \sum_{i=0}^{n} \alpha_i \xi(t_i) \tag{1.2}$$

where α_i is the coefficient which will be given in a particular formula [6]. One can obtain polynomials very close to the optimal form via expanding the given function based Chebyshev polynomials (CP) and then cutting off the expansion at the required degree which is roughly analogous to Fourier analysis, using the CP to substitute the usual trigonometric functions. If $P(t)$ in (1.2) is the CP of the second kind $U_n(t)$ and if we see $\xi = P_n$ at the points t_k, yields;

$$\xi(t_k) = \sum_{i=0}^{n} \alpha_i U_i(t_k), \qquad -1 \leq t \leq 1 \tag{1.3}$$

where

$$U_n(t) = \frac{\sin(n+1)\vartheta}{\sin\vartheta}, \quad \vartheta = \arccos t.$$

Hadid et al. [3] considered a generalized CP of the first type based ABC operator to describe the dynamics of the infection spreading of COVID-19 between infected and asymptomatic styles. They presented the symmetric solution with constant connections by using the sum CP of the first kind $T_n(t)$ to approximate the function that represent the infected people and the corresponding spreading phase. However the normal growth approaches have been tested to describe the time development of the COVID-19 infection [11].
The COVID-19 virus is appeared speedily in mostly all countries which then characterized as a pandemic by The World Health Organization (WHO). The first WHO register of dyed-in-the-wool testers of coronavirus identified to 282 cases on January 21, 2020 (see [15], [16]). Several growing approaches have been very newly involved in the time increase of the corona infection [11]. Mathematically, the following ordinary differential equation can translate the corona cases:

$$\frac{dg(t)}{dt} = g(t), \quad t \geq 0, \tag{1.4}$$

where g is the exponential function indicates the number of infected persons, the rampant phase, the rising number of asymptomatic infected persons are considered. Presently, there are various numerical studies and analytic surveys about corona have been attained (see [4], [10], [13], [14]).

Embedded research work investigates the numerical solution to COVID-19 problem based the hyper-singular integral (HSI). CP of second kind $U_n(t)$ is implemented to simulate data concerning the coronavirus. Our method designates a relationship in the model of integral equation (see [7, 8, 9]). We test the suggested approach by commissioning live data form Malaysia. Numerical experiments are introduced which underline the theoretical estimates. Moreover, the error function is applied to determine the compact interval of the infected number.

2. Methodology

The real function ξ in (1.1) is approximated by the orthogonal CP of the second kinds $U_n(t)$ which is well defined over [-1,1] [12]. The technique of the presented method is given as follows:

2.1. *The hypersingular integral*

Consider the problem in (1.1) is of Cauchy kernel and let the resulting Cauchy-type singular integral

$$\int_a^b \xi(t)dt = C\int_a^b \frac{g(t)}{t-\chi}\,dt \quad (\text{for } a<\chi<b)$$

exists, then it can be differentiated with respect to χ to get the Hadamard-type singular integral as follows:

$$\frac{d}{d\chi}\left(C\int_a^b \frac{g(t)}{t-\chi}\,dt\right) = \mathcal{H}\int_a^b \frac{g(t)}{(t-\chi)^2}\,dt \quad (\text{for } a<\chi<b). \tag{2.1}$$

Note that the symbols C and $\mathcal{H}$ are utilized here to indicate the Cauchy principal measure and Hadamard finite-part integrals accordingly.

The Hadamard finite part integral above (for $a<\chi<b$) can again be formulated by the following equal formulas:

$$\mathcal{H}\int_a^b \frac{g(t)}{(t-\chi)^2}\,dt = \lim_{\varepsilon\to0^+}\left\{\int_a^{\chi-\varepsilon}\frac{g(t)}{(t-\chi)^2}\,dt + \int_{\chi+\varepsilon}^b \frac{g(t)}{(t-\chi)^2}\,dt - \frac{g(\chi+\varepsilon)+g(\chi-\varepsilon)}{\varepsilon}\right\},$$

$$\mathcal{H}\int_a^b \frac{g(t)}{(t-\chi)^2}\,dt = \lim_{\varepsilon\to0^+}\left\{\int_a^b \frac{(t-\chi)^2 g(t)}{((t-\chi)^2+\varepsilon^2)^2}\,dt - \frac{\pi g(\chi)}{2\varepsilon} - \frac{g(\chi)}{2}\left(\frac{1}{b-\chi}-\frac{1}{a-\chi}\right)\right\}.$$

The above limit expressions replace the definition of Hadamard or such called Hyper-singular integral given in (2.1).

2.2. *Chebyshev polynomial of second kind (CP)*

The Chebyshev polynomials of the second kind (U_n) is formulated as follows:

$$U_n(\cos(\vartheta))\sin(\vartheta) = \sin((n+1)\vartheta).$$

These polynomials are indicated the solutions of Chebyshev differential equations

$$(1-t^2)\,y'' - 3\,t\,y' + n(n+2)\,y = 0\,.$$

The CP of the second kind is defined by the recurrence relation

$$\begin{aligned} U_0(t) &= 1\\ U_1(t) &= 2t\\ U_2(t) &= 4t^2-1\\ &\ \vdots\\ U_{n+1}(t) &= 2t\,U_n(t) - U_{n-1}(t)\,. \end{aligned}$$

In general, we have

$$\sum_{n=0}^{\infty} U_n(t)\,z^n = \frac{1}{1-2tz+z^2}\ ;$$

the exponential generating function is

$$\sum_{n=0}^{\infty} U_n(t)\frac{z^n}{n!} = e^{tz}\left(\cosh\left(z\sqrt{t^2-1}\right) + \frac{t}{\sqrt{t^2-1}}\sinh\left(z\sqrt{t^2-1}\right)\right).$$

2.3. The hypersingular integral of the Chebyshev polynomials of second kind

By using the technique in [2], we can consider the hypersingular integral of the CP of second kind. The technique is based on the collocation points

$$t_i = \cos(\frac{i}{n+1})\pi, \quad i = 1, 2, 3, ..., n,$$

where n represents the order of U_n. Now by using the hypersingular integral (2.1), we have

$$\mathcal{H}\int_a^b \frac{U_n(t)}{(t-\chi)^2}\,dt \approx \sum_{k=0}^{n-1} \alpha_k U_k,$$

where $\alpha_k \approx \dfrac{1}{1+k}$.

2.4. Error computations

The function $f(t)$ included in $C[a, b]$ (the space of continuous functions in the interval $[a, b]$) can be approximated as the truncate Chebyshev series of degree n :

$$f(t) \approx \frac{1}{2} f_0 + \sum_{i=1}^{n} f_i\, U_i(t),$$

where f_n are the constant coefficients. Let

$$P_n(t) = \frac{1}{2} f_0 + \sum_{i=1}^{n} f_i U_i(t),$$

then the error between the Chebyshev series U_n and the original function $f(t)$ is given as follow [5]

$$E_n f = \|f(t) - P_n(t)\| \le \frac{2^{-n}}{(n+1)!}\|f_{n+1}\|,$$

where $\|.\|$ indicates the n-dimensional Euclidean space $\mathbb{R}^n$, the intuitive notion of length of the vector $y = (y_1, y_2, ..., y_n)$ is captured by the formula

$$\|\mathbf{y}\|_2 := \sqrt{y_1^2 + \cdots + y_n^2}.$$

Fig.2.1 shows the error function of $U_2(t)$, where we use it to approximate our data. The root is determined such that $t \approx 0.5... \in [0, 1]$ satisfying the converge series around $t = 0$

$$E_n = -erf(1) + \frac{(8t^2)}{(e\sqrt{(\pi)})} + \frac{(16t^4)}{(e\sqrt{(\pi)})} + O(t^5)$$

with the global minimum

$$\min erf(4t^2 - 1) = -erf(1), \quad t = 0.$$

3. Results

Here, we attempt to test our scheme by signifying real data from the internet. Figure 1 indicates the cumulative monthly information (data) in Malaysia since the beginning of the pandemic. The figure is useful to understand the general growth behavior of the infection among the community.

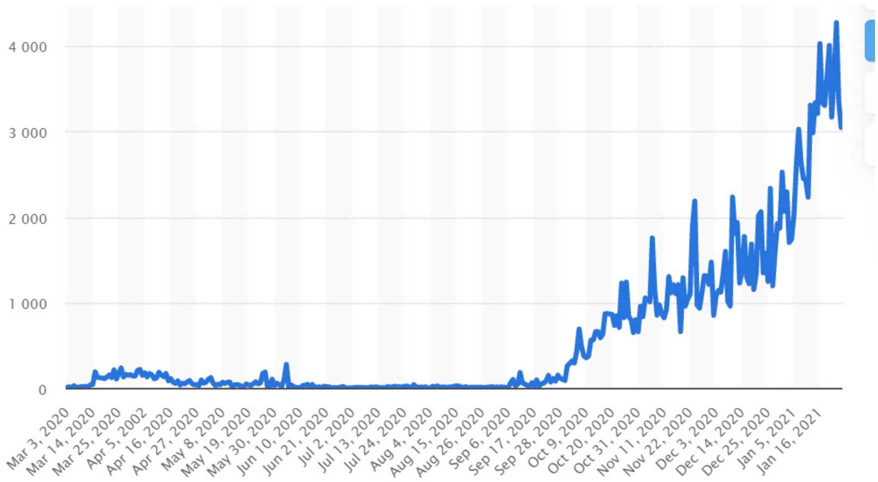

Figure 1. Infected numbers in Malaysia since March 2020 -January 2021

It is obvious that Figure 1 data of exponential form presented in (1.4). However this work particularly concerns on the numbers of infected people in October, November and December, see Figure 2. We utilized MATLAB software for statistical computation of the data.

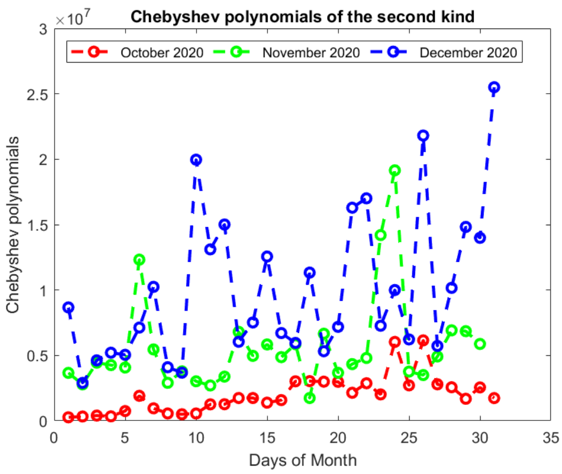

Figure 2. Chebyshev polynomials of the second kind without normalization for infected daily numbers

Our results explain how the CP lets the infected systems stay full-bodied in the ordinary, even when they determine periodic or chaotic performance. In other words, it proves that CP in inconsistency of external disturbances (parameter changes) can be preserved even when systems are irresolute. CP carries a full of life controlling device that covers beyond the stability of the approximated outcome and coded the system.

• The first step of the method is to normalize the data by using the formula

$$X_{normal} = \frac{x_{\max} - x}{x_{\max} - x_{min}}$$

to obtain data in the interval $x \in [0, 1]$. Another normalization of a huge data can be evaluated by using the Z-score function

$$X_{normal} = \frac{x - \mu}{\sigma},$$

where μ is the average and σ is the variance.

• The second step is to operate the normalized data by using CP. After several testings, we confirm that $U_2(t)$ covers our data.

• The root $t \approx 0.5... \in [0, 1]$ indicates that the most cases are clustering in the interval around t satisfying the interval

$$[0.5 - \epsilon, 0.5 + \epsilon], \quad \epsilon > 0.$$

The integration of the error around $t = 0.5 \approx 1/2$ is determine as follows:

$$I_t = |\int_0^{1/2} -erf(1 - 4t^2)dt| \approx 0.3073646887317632\cdots.$$

As a consequence, we can determine the value of ϵ to recognize the compact interval of the most infected people as follows:

$$\epsilon \approx t - I_t = 0.5 - 0.3073646887317632 \approx 0.19263531126.$$

Hence, the interval of the compact infected people converges to $[0.3, 0.7]$.

• Figures (3-5) show the actual normalized data vs the CP outcomes. These data are suggested for October, November and December, respectively.

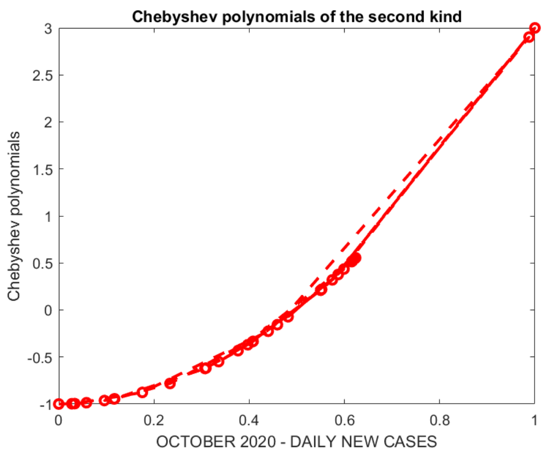

Figure 3. x-axis: Normalized data (circle), y-axis: CP of normalized data (dash) in October

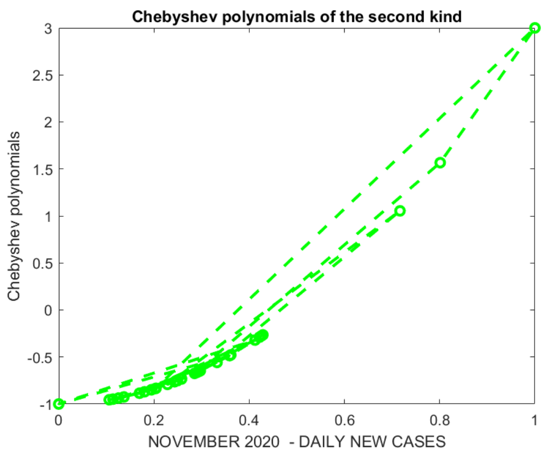

Figure 4. x-axis: Normalized data (circle), y-axis: CP of normalized data (dash) in November

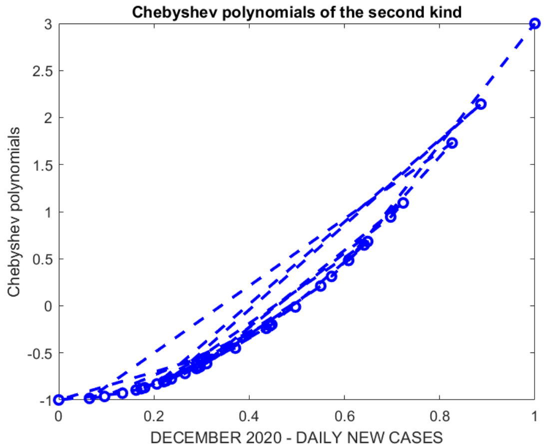

Figure 5. x-axis: Normalized data (circle), y-axis: CP of normalized data (dash) in December

4. Discussion and conclusion

In this place, we indicate that the usage of Chebyshev polynomial has the edibility to control the path of the outcome (data) for the infected system in COVID-19. The Chebyshev polynomial operation includes the organizer term with different kinds of network functions. The existence of the dynamical construction of the growth is given by utilizing self-function.

Moreover, the error function is determined for the second order Chebyshev polynomial (U_2) for the constructed data, and then we determined the root of the convergence followed by the interval of the compact infected data using the approximated law ϵ= root- integrator of the root.

The results confirmed that the strict government's movement control order (MCO) measures in Malaysia was significantly effective in reducing the direct contact rates among people, increasing the rate of detection and compulsory isolation or quarantine of the infectious cases. MATLAB code is developed to obtain the numerical results for the proposed problem, where the numerical examples assert the theoretical results and we found agreement action on the graphs.

This study can be applied for different data of COVID-19 depending on the suitable selection of the order of Chebyshev polynomial. In our next study we shall employ the concept of fractional calculus to present a generalization of Chebyshev equation to evaluate a fractional Chebyshev polynomials.

5. Appendix

In this section, we give the code of the computation. The first part is the normalization computation and the second part is the normalized data under the hyper-integral of CP (U_2).

$$AN = normalize(A,' range')$$
$$xn1 = AN(1:31,1); \textit{first column-Oct}$$
$$xn2 = AN(1:30,2); \textit{first column- Nov}$$
$$xn3 = AN(1:31,3); \textit{first column- Dec}$$
$$OctN = (Oct - min(Oct))/(max(Oct) - min(Oct));$$
$$NovN = (Nov - min(Nov))/(max(Nov) - min(Nov));$$
$$DecN = (Dec - min(Dec))/(max(Dec) - min(Dec));$$
$$n = 2;$$
$$yn1 = chebyshevU(n, xn1);$$
$$yn2 = chebyshevU(n, xn2);$$
$$yn3 = chebyshevU(n, xn3);$$

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Data and material used in this paper is available online at https://www.statista.com/statistics/1110785/malaysia-covid-19-daily-cases/.

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