Montes Taurus J. Pure Appl. Math. 4 (1), 142–148, 2022

Montes Taurus Journal of Pure and Applied Mathematics
https://www.mtjpamjournal.com

MTJPAM
ISSN: 2687-4814

Investigation of bauxite module classification using regression analysis and inequality expressions involving them

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Abstract

In engineering applications, the classifications of ore quality are significant since this form of application plays a key role in the use of ore in industry. This study determines the quality of Minim-Martap bauxite ore based on the modulus. The modulus of Al_2O_3/SiO_2 was used for this classification. First of all, the composition of the bauxite was determined and the correlation between the contents was considered. The analysis was optimized by introducing iterations, and the correlation between the contents was revealed using the regression analysis. Then, the modulus classification and/or quality of bauxite was determined using the inequality expressions.

Keywords: Optimization, inequality expressions, bauxite quality classification, bauxite modulus (Al_2O_3/SiO_2), regression analysis, applied mathematics

2010 MSC: 00A69, 62J05, 62H25, 62-07, 60E15, 62H11

1. Introduction

Bauxite ore, which is considered among the valuable metals, is processed to obtain aluminum. The chemical contents of the bauxite ore are taken into account to determine the quality of the ore. Among these contents, particularly the compounds of Al_2O_3 and SiO_2 reveal the quality of bauxite ore. In the literature, there are several studies, classifications, and bauxite modules for determining the quality of bauxite [6, 8, 9, 12, 14, 19, 31, 34, 35, 39]. Besides, several researchers tried to solve some scientific problems by using inequality expressions and regression analyses in many fields in recent years. Scientific studies involving various inequality expressions about mines have gained prominence in general [1, 7, 10, 13, 15, 16, 21, 23, 28, 30, 32, 37].

The inequality expressions and regression analyses related to bauxite, which is the subject of the study, have been increasingly gaining prominence each passing day [2]-[4], [11, 17, 18, 22, 25, 26, 27, 29, 33, 36, 40, 42, 44, 47]. However, in the literature, there is no detailed scientific study on the inequality expressions describing the quality of bauxite. Collecting data about the contents of bauxites, using these data, and interpreting them is important. Moreover, determining the inequalities revealed by obtaining the data, the procedure used to obtain these inequalities, and types of data asymmetries in the data becomes significant. These data can facilitate several processes in the mining industry, mining companies, and governments interested in these mines. The present study aims to classify Minim-Martap bauxite samples with different industrial chemical properties, to determine the correlation between

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Received:20 December 2021, Accepted:14 January 2022, Published:9 February 2022

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aluminum and silica, and to describe the quality of bauxite using inequality expressions. In the classification study, the ratio between the Al_2O_3 and SiO_2 values (Al_2O_3/SiO_2) obtained in chemical analyses was taken into account, regression analyses were performed. Then, each class, which was determined according to these data, was defined as a different modulus. Each modulus used in the classification was separately evaluated, optimized, and interpreted by defining an equality or inequality expression.

2. Methodology

The chemical analyses reveal the contents of the bauxite samples. The oxide contents of the ore allow identifying the ore and the proportions of these oxides reveal the quality of the ore. Bauxite samples from the Minim-Martap Plateau were examined in the study [25]. Table 1 presents the details about the chemical contents of the bauxite samples. Regression analysis was conducted using these data. According to the results of the analysis, which was conducted using the SPSS statistical software package, the regression equation was obtained.

Minim-Martap	SiO_2	Al_2O_3	Fe_2O_3	CaO	MgO	Na_2O	K_2O	MnO	TiO_2	P_2O_5	Cr_2O_3	Ba	LOI	Total
MB25	0.65	58.81	3.94	0.02	0.02	0.01	0.01	0.02	4.35	0.10	0.09	0.02	31.54	99.56
MB26	1.84	58.93	3.25	0.02	0.04	0.01	0.01	0.01	3.75	0.11	0.06	0.03	31.32	99.37
MB27	1.16	54.77	4.72	0.16	0.05	0.01	0.01	0.02	6.11	1.19	0.07	0.27	29.78	98.36
MB28	13.74	42.06	18.11	0.02	0.02	0.01	0.01	0.02	2.09	0.20	0.09	0.02	22.90	99.26
MB29	1.00	60.03	3.07	0.02	0.04	0.01	0.01	0.01	3.06	0.08	0.07	0.02	31.87	99.25
MB30	1.97	54.90	5.88	0.04	0.04	0.01	0.01	0.02	6.04	0.25	0.06	0.07	29.78	99.07
MB31	0.49	58.88	3.55	0.01	0.02	0.01	0.01	0.01	4.76	0.10	0.11	0.02	31.50	99.44
MB32	1.25	58.77	3.72	0.03	0.02	0.01	0.01	0.02	3.79	0.15	0.07	0.03	31.37	99.22
MB33	6.71	55.66	3.67	0.02	0.02	0.01	0.01	0.01	3.35	0.13	0.07	0.03	29.29	98.98
MB34	2.30	52.64	9.75	0.03	0.02	0.01	0.01	0.03	4.40	0.23	0.08	0.05	29.47	99.01
MB35	1.17	58.41	4.27	0.02	0.03	0.01	0.01	0.02	4.29	0.11	0.12	0.02	30.99	99.46
MB36	0.92	55.41	7.43	0.02	0.03	0.01	0.01	0.02	5.27	0.10	0.10	0.02	30.05	99.36
MB37	2.35	52.85	8.62	0.01	0.04	0.01	0.01	0.02	5.60	0.10	0.14	0.01	29.64	99.38
MB38	1.18	56.58	4.92	0.02	0.03	0.01	0.01	0.02	5.48	0.17	0.10	0.04	30.96	99.49
MB39	1.69	55.09	6.70	0.01	0.03	0.01	0.01	0.01	5.38	0.12	0.09	0.02	30.50	99.63
MB40	0.68	57.91	4.23	0.01	0.02	0.01	0.01	0.01	4.96	0.10	0.13	0.02	31.46	99.51
MB41	2.30	41.04	26.03	0.01	0.03	0.01	0.01	0.15	4.57	0.18	0.13	0.01	24.99	99.45
Mean	2.44	54.87	7.17	0.03	0.03	0.01	0.01	0.02	4.54	0.20	0.09	0.04	29.85	99.28

Table 1. Results of the chemical analysis of the bauxite samples [25]

2.1. Al_2O_3/SiO_2 modulus ratio

The main content of the bauxite is aluminum. The proportions of the aluminum (Al_2O_3), silicate (SiO_2), iron (Fe_2O_3), titanium oxides (TiO_2), and calcium oxide CaO are of great importance for the bauxite. The proportions of the contents of the bauxite facilitate classifying bauxite [25, 38, 30]. The calculation of the (Al_2O_3/SiO_2) modulus ratio in the bauxite and the scarcity and abundance of the contents describe the quality of the bauxite [38]). Using this calculation method, bauxite samples were classified based on their different ratios; thereby, the ore quality of the bauxite sample was determined (Table 2).

Al_2O_3/SiO_2 content (%)	Classification of bauxite
$Al_2O_3/SiO_2>20$	High alumina ore
$Al_2O_3/SiO_2=11\text{-}20$	Alumina ore
$Al_2O_3/SiO_2=4\text{-}10$	Siliceous ore
$Al_2O_3/SiO_2<4$	High siliceous ore

Table 2. Classification of the bauxite using the (Al_2O_3/SiO_2) modulus [38, 24]

2.2. Regression analysis

Regression analysis, which is widely used in engineering applications, is used to model the dependent variable, particularly in statistical studies. Furthermore, in the event that the number of independent variables is more than one, then multiple regression analysis can be conducted using the SPSS software package [5, 30, 41], [43]-[46]. The results of the multiple regression analysis are presented by Modal Summary, ANOVA and the regression coefficients based on the number of independent variables. Also the confidence interval was determined based on the statistic.

2.3. Inequality

While considering the inequalities and asymmetries related to the data, new ideas should be generated regarding a better understanding of the issue under consideration [21, 20]. In this context, it was decided which data to be used and how they will be used; then, the inequalities to be used in their calculation were determined.

3. Main results and discussion

3.1. Regression analysis

The results of the regression analysis of the Minim-Martap Bauxite samples are given in Table 3. Regression analysis was performed according to SiO_2. The Durbin–Watson test statistic was used to see whether the data is correlated, and the correlation coefficient was calculated as 1.416. According to these data, it was found that there was no autocorrelation. The significant value of the ANOVA was found to be 0.000, which implied that it could be used in the statistical analysis. Likewise, the R^2 value was found to be 99%, and the confidence interval for the coefficients was calculated as (F:94% confidence level). According to the results of the analysis, the data used in the analysis were found to be highly significant. The regression equation was obtained based on the contents of the bauxite. Then, the SiO_2 modulus was determined based on this equation. While all other variables are kept constant, one unit increase in the Al_2O_3 content was found to cause a 0.782 unit decrease in the value of the SiO_2 content. Accordingly, the equation was obtained as follows (3.1):

$$\begin{aligned} Modulus(SiO_{2\text{Minim-Martap}}) &= 92.477 - 0.782Al_2O_3 - 0.895Fe_2O_3 - 20.317CaO \\ &\quad +0.777MgO - 5.180MnO - 0.825TiO_2 + 5.042P_2O_5 - 4.481Cr_2O_3 \\ &\quad -20.805Ba - 1.208LOI. \end{aligned} \tag{3.1}$$

Model Summary[b]

Model	R	**R Square**	Adjusted R Square	Std. Error of the Estimate	Change Statistics: R Square Change	F Change	df1	df2	Sig. F Change	**Durbin-Watson**
1	1.000[a]	**0.999**	0.998	0.13342	0.999	942.282	10	7	0.000	**1.416**

a. Predictors: (Constant), LOI, CaO, Cr_2O_3, MgO, MnO, TiO_2, Fe_2O_3, P_2O_5, Al_2O_3, Ba

b. Dependent Variable: SiO_2

ANOVA[a]

Model		Sum of Squares	df	Mean Square	**F**	Sig.
1	Regression	167.732	10	16.773	**942.282**	0.000[b]
	Residual	0.125	7	0.018		
	Total	167.856	17			

a. Dependent Variable: SiO_2

b. Predictors: (Constant), LOI, CaO, Cr_2O_3, MgO, MnO, TiO_2, Fe_2O_3, P_2O_5, Al_2O_3, Ba

Coefficients

Model		Unstandardized Coefficients: B	Std. Error	Standardized Coefficients: Beta	t	Sig.	Correlations: Zero-order	Partial	Part
1	(Constant)	92.477	2.918		31.692	0.000			
	Al_2O_3	-0.782	0.098	-1.322	-7.936	0.000	-0.638	-0.949	-0.082
	Fe_2O_3	-0.895	0.060	-1.684	-14.979	0.000	0.463	-0.985	-0.154
	CaO	-20.317	10.355	-0.220	-1.962	0.091	-0.085	-0.596	-0.020
	MgO	0.777	4.580	0.002	0.170	0.870	-0.278	0.064	0.002
	MnO	-5.180	3.273	-0.052	-1.582	0.158	0.002	-0.513	-0.016
	TiO_2	-0.825	0.083	-0.278	-9.983	0.000	-0.627	-0.967	-0.103
	P_2O_5	5.042	2.734	0.404	1.844	0.108	-0.028	0.572	0.019
	Cr_2O_3	-4.481	3.206	-0.033	-1.398	0.205	-0.101	-0.467	-0.014
	Ba	-20.805	11.780	-0.391	-1.766	0.121	-0.109	-0.555	-0.018
	LOI	-1.208	0.137	-0.895	-8.815	0.000	-0.791	-0.958	-0.091

Table 3. Regression analysis

The regression equation obtained according to SiO_2 revealed that there was a negative relationship between SiO_2 and Al_2O_3. In the event that one of them increases, the other decreases or vice versa, in the event that one decreases, the other increases. Therefore, the ratio between Al_2O_3 and SiO_2 has gained importance for the inequality expressions/equations used while determining the bauxite quality.

3.2. Inequality expressions

The equation for defining an ore sample as bauxite is as follows (3.2):

$$100 \text{ units} = Al_2O_3 + SiO_2 + Fe_2O_3 + TiO_2 + CaO + \text{Other Oxides} + LOI \text{ (loss on ignition)}. \quad (3.2)$$

Particularly, the Al_2O_3/SiO_2 ratio in the chemical content of the bauxite can be used as the bauxite modulus (M). Taking into account the inequality and equality expressions, the modules of the bauxite class with different properties are described in four groups. Considering the aluminum ratio from high to low, these modules are defined as "high alumina ore (HAO)", "alumina ore (AO)", "siliceous ore (SO)" and "high siliceous ore (HSO)", respectively [38, 24]. The modules determined based on the chemical contents of bauxite samples are calculated using the following equation (3.3):

$$\text{Modulus}(M) = Al_2O_3/SiO_2. \quad (3.3)$$

The calculated values of the modules indicate different definitions of bauxite. According to the inequality expressions, the first modulus is calculated as "high alumina ore (HAO)", and its equation is as follows (3.4):

$$M_{\text{HAO}} > 20. \quad (3.4)$$

Considering the equation, "alumina ore (AO)" and "siliceous ore (SO)" have different definitions. The following inequality expressions differentiate the quality of bauxite in these modules (3.5) and (3.6):

$$11 \leq M_{\text{AO}} \leq 20, \quad (3.5)$$

$$4 \leq M_{\text{SO}} \leq 10. \quad (3.6)$$

The last inequality expression, which has the lowest bauxite content, is defined as "high siliceous ore (HSO)" (3.7).

$$M_{\text{HSO}} > 4. \quad (3.7)$$

The Al_2O_3/SiO_2 ratios of the chemical contents of the Al_2O_3 and SiO_2, i.e. the modules, were calculated (Table 4). The average modulus ratio was found to be 22.48 for the Minim-Martap samples. Accordingly, the sample group was defined as "high alumina ore, HAO". While Sample 7 (120) was found to have the maximum modulus value, Sample 4 (3.6) was found to have the minimum modulus value (Figure 1).

No	Minim-Martap Samples	Modulus
1	MB25	90.5
2	MB26	32
3	MB27	47.2
4	MB28	3.06
5	MB29	60
6	MB30	27.9
7	MB31	120
8	MB32	47
9	MB33	8.3
10	MB34	22.9
11	MB35	49.9
12	MB36	60.2
13	MB37	22.5
14	MB38	47.95
15	MB39	32.6
16	MB40	85.2
17	MB41	17.8

Table 4. Modulus values of the bauxite samples

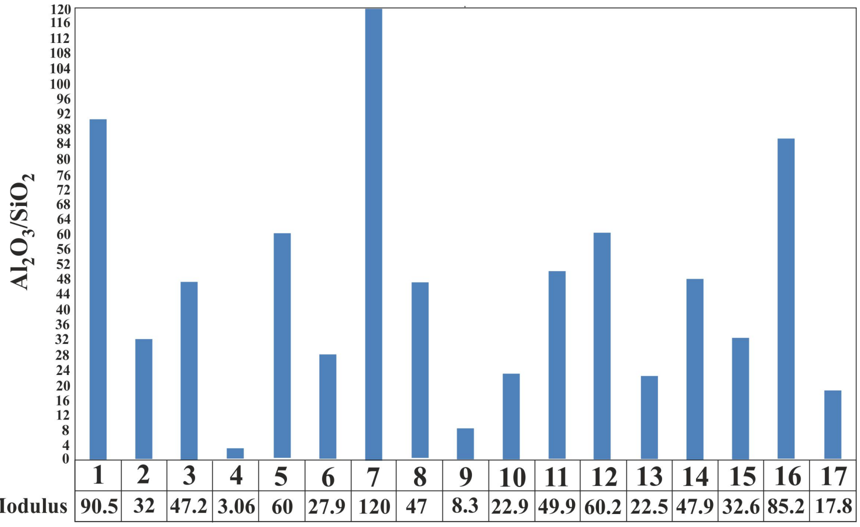

Figure 1. Histogram of the modulus values and the threshold values

4. Conclusions

The Durbin-Watson test statistic (1.416) revealed that there was no autocorrelation in the data and the Sig. value of ANOVA (0.000) implied that this dataset can be used in the statistical analysis. Also, the R^2 value (95.8%) and the confidence interval of the coefficients (F: 94%) were found to be highly significant. The multiple regression formula was obtained for the bauxite samples, and the SiO_2 modulus was found to be one of the significant variables. In particular, when all other variables contained by the bauxite, except for SiO_2, are kept constant, an increase of one unit of Al_2O_3, which is the most significant content of the bauxite, causes a decrease in the SiO_2 by 0.782 units. The equation of regression analysis reveals the relationship between the SiO_2 and the Al_2O_3 variables. In this case, the regression equation is determined as follows (4.1):

$$\begin{aligned} \text{Modulus}(SiO_2) &= A + E_{\text{MgO}} + HP_2O_5 - BAl_2O_3 - CFe_2O_3 - DAl_{\text{CaO}} \\ &\quad -FAl_{\text{MnO}} - GTiO_2 - ICr_2O_3 - JAl_{\text{Ba}} - KAl_{\text{LOI}}. \end{aligned} \tag{4.1}$$

The bauxite modulus (M) based on the Al_2O_3/SiO_2 ratio was found to be high alumina ore (HAO) for 14 samples, alumina ore (AO) for 1 sample, siliceous ore (SO) for 1 sample, and high siliceous ore (HSO) for 1 sample. According to these figures, this ore has high quality. Regression analysis can be used to determine the source and the quality of the bauxite ore, and the modulus calculation can be easily used to determine the quality of bauxite. The bauxite modulus calculations were conducted by taking into account the inequality expressions for the bauxite ores, and the regression analysis was successfully used.

Acknowledgments

The author would like to thank Daniel G. NYAMSARI at Akdeniz University for his assistance with the chemical data interpretation of the samples.

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