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Editorial

Special Issue: Recent Trends on Pure and Applied Mathematics involving Nonlinear Analysis, Global Analysis, Approximation Theory, Functional Analysis, Functional Equations, and Inequalities and their Applications "Dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday"

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Abstract

This paper presents brief summaries of the articles written on the topic of Pure and Applied Mathematics and published in the Special Issue: Recent Trends on Pure and Applied Mathematics involving Nonlinear Analysis, Global Analysis, Approximation Theory, Functional Analysis, Functional Equations, and Inequalities and their Applications. Since this special issue is dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday, this paper also includes a brief biography of Professor Themistocles M. Rassias.

Keywords: Pure and Applied Mathematics, Mathematics in general, General applied mathematics, Nonlinear Analysis, Global Analysis, Approximation Theory, Functional Analysis, Functional Equations, Inequalities.

2010 MSC: 00A05, 00A69.

The topics covered under the title "Recent Trends on Pure and Applied Mathematics involving Nonlinear Analysis, Global Analysis, Approximation Theory, Functional Analysis, Functional Equations, and Inequalities and their Applications" play an important role in many branches of Mathematics, Statistics and allied areas such as Engineering, Computer Science, Physics and Mathematical Physics, Economics, and also other related areas. This Special Issue deals with the theory and applications of Pure and Applied Mathematics, especially the keywords for this special

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issue are *Nonlinear Analysis, Global Analysis, Approximation Theory, Functional Analysis, Functional Equations, Inequalities and their Applications, Real and Complex Analysis, Integral Equations and Integral Transforms, Higher Transcendental Functions and Their Applications, q-Series and q-Polynomials, Analytic Number Theory, Analytic and Geometric Inequalities, Probability and Statistics, Inventory Modeling and Optimization, Ordinary Differential Equations, Difference Equations, Partial Differential Equations, Fractional Differential Equations, Stochastic Differential Equations, Time-Scale Dynamic Equations, Related Topics on Differential Equations*.

This Special Issue is dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday, in recognition of his significant contributions in many different areas of Pure and Applied Mathematics.

As a co-worker and collaborator, Professor Rassias is very kind, respectful, and genuinely helpful to almost everyone. Professor Rassias is featured in written documents and websites where he has made outstanding contributions to hundreds of people, as well as thousands of single-authored and co-authored scientific journal articles, books and edited volumes in Pure and Applied Mathematics. For this reason, issues of journals and special books have been dedicated to Professor Rassias so far.

We, as editors, are honored and delighted to contribute to the preparation of this Special Issue dedicated to Professor Rassias for its publication in the "Montes Taurus Journal of Pure and Applied Mathematics". We believe with our deepest feelings that, since our journal "Montes Taurus Journal of Pure and Applied Mathematics" aims to unite all scientists in the world and to "keep serving humanity" forever without compromising its scientific policy, not only this special issue. All of the other volumes of this journal will always serve all researchers dealing with scientific researches and endeavors.

1. Special Issue Information:

All manuscripts of this Special Issue in the *Montes Taurus Journal of Pure and Applied Mathematics* are dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday. Professor Rassias's scholarship, global impact, professional contributions, and many friendships and collaborations deserve celebrating! We, as editors, are appreciative to be able to write this article in Professor Themistocles M. Rassias's honor. We are also honored to thank him most sincerely for his remarkably outstanding contributions to many different areas of Pure and Applied Mathematics.

This Special Issue aims to deal essentially with the theory and applications of the areas developed and advanced by **Professor Themistocles M. Rassias** and his research collaborators. Especially, we invited high-quality original research papers, as well as survey papers related to the topics of this Special Issue for possible publication in this Special Issue.

As for the brief curriculum vitae of **Professor Themistocles M. Rassias**:

Themistocles M. Rassias was born on April 2, 1951 in Pellana, Lakonia, Greece. He has published more than 60 books, monographs and edited volumes, as well as more than 350 peer-reviewed research journal articles. Additionally, his work has received around 17000 citations in Google Scholar with h-index 47 and more than 5300 in MathSciNet. He has memberships of Editorial Board of many international journals. His work extends over several fields of Mathematical Analysis. It includes Global Analysis, Analysis on Manifolds, Calculus of Variations, Nonlinear Functional Analysis, Approximation Theory, Functional Equations, Inequalities, Metric Geometry and their Applications. Further details about Professor Themistocles M. Rassias' academic activities, achievements, as well as honors, awards and distinctions, can be found at the following website:

- `https://en.wikipedia.org/wiki/Themistocles_M._Rassias`

2. Special Issue Overview:

The Special Issue contains 19 contributions in Pure and Applied Mathematics. We now highlight the main results involving abstracts of the following manuscripts:

- In [1], the authors introduce and consider a new class of variational inequalities, which is called the nonconvex bifunction general variational inequality. Using the auxiliary principle technique, they suggest and analyze some iterative methods for solving the nonconvex bifunction general variational inequalities. They prove that

the convergence of these methods either requires only pseudomonotonicity or partially relaxed strongly monotonicity. Their proofs of convergence are very simple. The ideas and techniques of the paper [1] may stimulate further research in this field.

- The Covid-19 pandemic is a major healthcare disaster of global proportions, and has resulted in immense suffering and loss of life, along with disruptions in economic and social activities. Governments of many nations in the midst of the pandemic have been instituting a variety of trade policies, including tariffs and quotas, on products deemed important to their citizenry. Many of the products, such as food items, medicines, and even PPEs are perishable products. In [2], the author construct the first multiproduct spatial price equilibrium model, in both static and dynamic versions, that captures product perishability, and trade policies in the form of tariffs and quotas. The static model is formulated as a variational inequality problem and the dynamics studied using a projected dynamical system. Theoretical results are presented along with a series of increasingly complex numerical examples, the solutions to which are computed using a proposed algorithm, for which convergence results are also given. The results illustrate the importance of having a rigorous theoretical and algorithmic framework to assess the impacts of trade policies, before their implementation.

- Let $a, b, c \in \mathbb{R}^2$, $a_i < c_i < b_i$, $i \in \{1, 2\}$, $[a, b] := [a_1, b_1] \times [a_2, b_2]$, let $(\mathbb{B}, |\cdot|)$ be a (real or complex) Banach space, $K \in C([a, b] \times [a, c] \times \mathbb{B}, \mathbb{B})$, $H \in C([a, b] \times [a, b] \times \mathbb{B}, \mathbb{B})$ and $g \in C([a, b], \mathbb{B})$. In [3], the authors study the following integral equation

$$u(x) = \int_{[a,c]} K(x, s, u(s))ds + \int_{[a,x]} H(x, s, u(s))ds + g(x),$$
$$x = (x_1, x_2) \in [a, b].$$

  Using the Fibre Contraction Principle we give existence and uniqueness results, and they prove the convergence of the successive approximations. By the weakly Picard operator theory (in the framework of the ordered Banach space $\mathbb{B}$) they give Gronwall lemma type results and comparison theorems. Some other similar type of Fredholm-Volterra integral equations are also studied.

- There are different investigations unfilled the population energetic of COVID-19. In [4], the authors formulate an approximated hypersingular integral based Chebyshev polynomials of second kind to simulate COVID-19 growth. The planned scheme indicates an association consequences of integral equation model by employing live data from Malaysia for three different months. MATLAB code is developed to obtain the numerical results for the presented problem. Moreover, the error function is applied to determine the compact interval of the infected number. The authors of [4] state that they could establish agreement action on the displays where the numerical results assert the theoretical concept.

- Numerous decomposition formulas for various hypergeometric functions of several variables have been offered. In [5], the authors aim to establish symbolic operator identities and decomposition formulas for second-order quadruple Gaussian hypergeometric series associated with Appell functions and Saran hypergeometric functions by mainly using mutually inverse symbolic operators $H(\alpha, \beta)$ and $\overline{H}(\alpha, \beta)$, which were introduced in an earlier work. Mellin-Barnes type contour integrals are employed for proofs of the operator identities. Also the authors determine the regions of convergence of the 14 quadruple Gaussian hypergeometric series.

- The aim of [6] is to give some new classes of finite sums involving the numbers $y(m, \lambda)$, the generalized harmonic functions, special numbers and polynomials, the Dedekind sums, and other combinatorial sum. Reciprocity laws for these sums are proven. Some applications of these reciprocity laws are presented. With aid of the reciprocity law of the Dedekind sums, formulas for many new finite sums are obtained. Relations among these new classes of finite sums, partial sum of the generalized harmonic functions, the Riemann zeta function, the Hurwitz zeta function, hypergeometric series, polylogarithms, digamma functions, polygmamma functions, and special numbers and polynomials and other combinatorial sums are given. Moreover, some formulas for the partial sum of the generalized harmonic functions and special numbers and polynomials are given. Finally, comments and observations on the results of [6] are given.

- In the study [7], the author introduce a solvability of special type of symmetric algebraic differential equations (SADEs) in virtue of geometric function theory by considering a symmetric differential operator. The analytic solutions of the SADEs are considered by utilizing the Caratheodory functions joining the subordination concept. A class of Caratheodory functions involving special functions gives the upper bound solution.

- In [8], the author introduce some new fractional integral operators and fractional area balance operators in the Banach spaces. The corresponding norm inequalities are established. They are significant improvement and generalizations of many known and new classes of fractional integral operators.

- The paper [9] deals with a class of strongly continuous semigroups generated by operators defined on the tensor product of Hilbert spaces. Explicit exponential stability conditions for the considered semigroups are derived. Applications of the obtained conditions to semigroups generated by matrix differential operators and integro-differential operators are also discussed.

- The authors of [10] compute the motive of the variety of representations of the torus knot of type (m, n) into the affine groups $\mathrm{AGL}_1(\mathbf{k})$ and $\mathrm{AGL}_2(\mathbf{k})$ for an arbitrary field $\mathbf{k}$. In the case that $\mathbf{k} = \mathbb{F}_q$ is a finite field this gives rise to the count of the number of points of the representation variety, while for $\mathbf{k} = \mathbb{C}$ this calculation returns the E-polynomial of the representation variety. The authors of [10] discuss the interplay between these two results in sight of Katz theorem that relates the point count polynomial with the E-polynomial. In particular, the authors of [10] shall show that several point count polynomials exist for these representation varieties, depending on the arithmetic between m, n and the characteristic of the field, whereas only one of them agrees with the actual E-polynomial.

- In [11], the author present the fixed point theorem for set-valued contraction mappings in generalized b-metric spaces, which generalizes the famous Nadler's fixed point theorem for such mappings in metric spaces. Also some local fixed point theorems for such multi-valued mappings are presented.

- Using convexity, ψ-uniformly convexity, N-quasiconvexity and N-quasisuperquadracity, the author of [12] extend and refine inequalities related to the Euler-Lagrange identity.

- In [13], A (Matkowski type) functional extension – to the realm of ordered metric spaces –is given for the diagonal fixed point result in Ćirić and Prešić [Acta Math. Comenianae, 76 (2007), 143-147] involving Prešić iterative processes.

- In [14], orthogonal polynomials related to Abel and Lindelöf weight functions on $\mathbb{R}$, as well as ones related to some products of these weight functions, are considered. Using the moments of the weight functions, the coefficients in the three-term recurrence relations are determined in the explicit form. Also, some connections with Meixner-Pollaczek polynomials with real parameters are presented.

- Let S be a semigroup, and let $\mu : S \longrightarrow \mathbb{C}$ be a multiplicative function such that $\mu(x\sigma(x)) = 1$ for all $x \in S$. In [15], the auhtors study the properties of the solutions of the functional equations

$$f(xy) + \mu(y)f(x\sigma(y)) = 2f(x)g(y) + 2f(y)g(x), \ \ x, y \in S,$$

$$f(xy) + \mu(y)f(x\sigma(y)) = 2f(x)f(y) + 2g(x)g(y), \ \ x, y \in S,$$

  where σ is an involutive morphism. The solutions are expressed by means of solutions of d'Alembert's μ-functional equation and the functional equation

$$f(xy) + \mu(y)f(x\sigma(y)) = 2f(x)\phi(y) + 2f(y)\phi(x), \ \ x, y \in S,$$

  in which ϕ is a solution of d'Alembert's μ-functional equation. As an application the authors prove that, in a nilpotent group G which is generated by its squares, the solutions of the functional equation

$$f(xy) + \mu(y)f(xy^{-1}) = 2f(x)g(y) + 2f(y)g(x), \ \ x, y \in G$$

  are abelian.

The authors of [15] also find the solutions of the functional equation

$$f(xy) + \mu(y)f(x\sigma(y)) = 2f(x)\phi(y) + 2f(y)\psi(x),\ \ x, y \in S$$

where σ is an involutive anti-automorphism, $f : S \longrightarrow \mathbb{C}$ is the unknown function and ϕ, ψ are non-zero solutions of d'Alembert's μ-functional equation. This enables us to solve the pexider functional equation

$$f(xy) + \mu(y)f(x\sigma(y)) = 2g_1(x)h_1(y) + 2\psi(x)h_2(y),\ \ x, y \in S$$

in which $f, g_1, h_1, h_2 : S \longrightarrow \mathbb{C}$ are the unknown functions and g_1 is even.

- In [16], the auhtor present some generalization of the Paluszyński, Stempak method of producing an "induced" metric by a b-metric, by using Cauchy multiplicative functional equation.

- In [17], the auhtors prove the Hyers-Ulam stability of general second-order linear differential equations by using Mahgoub integral transform method. Furthermore the auhtors provide some examples to illustrate main results.

- Since its genesis, an equation of Pexider type has captivated the attention of the mathematical fraternity around the world. Over the decades, several Pexiderized forms of various functional equations have been studied meticulously. In comparison to the functional equations, such forms are less analysed for sum form functional equations and require substantial study. Taking lead from it, the paper [18] is devoted to obtain the general solution of some Pexiderized forms of a sum form functional equation

$$\sum_{i=1}^{n}\sum_{j=1}^{m} T\left(p_i q_j\right) = \sum_{i=1}^{n} T\left(p_i\right)\sum_{j=1}^{m} T\left(q_j\right) + (m-n)T(0)\sum_{j=1}^{m} T\left(q_j\right) + m(n-1)T(0),$$

where T is a real-valued mapping with the domain $I = [0, 1]$; $(p_1, \ldots, p_n) \in \Gamma_n$, $(q_1, \ldots, q_m) \in \Gamma_m$ and $n \geq 3$, $m \geq 3$ are fixed integers.

- In [19], the authors introduce hom-derivations in complex Banach algebras. Using the fixed point method and the direct method, the authors prove the Hyers-Ulam stability of hom-derivations in complex Banach algebras, associated with the bi-additive s-functional inequality

$$\begin{aligned}&\|f(x+y, z-w) + f(x-y, z+w) - 2f(x, z) + 2f(y, w)\| \\ &\leq \|s\left(2f\left(\frac{x+y}{2}, z-w\right) + 2f\left(\frac{x-y}{2}, z+w\right) - 2f(x, z) + 2f(y, w)\right)\|,\end{aligned}$$

where s is a fixed nonzero complex number with $|s| < 1$.

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