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Topological indices of some operations in cycle graphs related to EV and VE degrees

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Abstract

Topological indices are numerical variables through which we can derive a relationship between molecular structure of a compound and its physical or biological properties. First step of this process is to represent the molecular structure of the compound as a mathematical graph and then derive degree or distance based topological indices. There are many topological indices derived so far in chemical graph theory and many of them have established a strong relationship with properties of some compounds. This article gives estimation and comparison of some of such indices for graphs derived from simple cycle graphs through graph operations such as corona product, single edge connected graph and rooted product graph. Indices are based on VE and EV degree, which are recently added to the chemical graph theory.

Keywords: VE-EV degree, corona product, single edge connected graph, rooted product graph

2020 MSC: 05C35, 05C07, 05C40

1. Introduction

Chemical graph theory is a subdivision of mathematical chemistry in which the fundamentals of graph theory applied on the molecular structure of a chemical compound and study the correlation between its physical property and the structure (*cf.* [2, 4, 12, 16]). Firstly we generate the graph from the chemical structure by identifying atoms as vertices and chemical bonds between the atoms as edges. Further this graph is reduced to a numerical descriptor called indices. Representation of molecular structure into mathematical graphs are widely use in chem-informatics (*cf.* [13, 15]). Establishing a correlation between mathematical descriptors and different physical and chemical properties of the compound is the basic concept use in QSAR and QSPR studies (*cf.* [7, 8, 12]). One of such widely quoted index is Wiener Index, and is defined as the sum of shortest path in a graph and this index has ability to predict boiling point of Paraffin.

Aside from chemical indices, graph theory applications in chemistry cover a wide range of problems like isomer enumeration, searching for molecular substructures in chemical databases. The connection between graph theory and chemistry has evolved significantly over time. The scientific field known as chemical graph theory was created as a result of several investigations relating to both fields (*cf.* [3]).

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1.1. Preliminaries

In this session, we give basic concepts and definitions. A graph which is represented as H having set of vertices $V(H)$ & set of edges $E(H)$. e denotes the edge connecting two vertices say u and v also $d(u)$ denotes the cardinality of adjacent vertices it shares.

Many bigger molecular structure of compounds can be assumed the combination of smaller units in which long chain compounds are typical examples. Studies on graphs obtained through different operations of simple graphs always imply keen interest among researchers in applied sciences (*cf.* [1, 9], [13]-[15]).

Definition 1.1. A graph is produced by taking one copy of G and $|V(G)|$ copies of H and attaching the i^{th} vertex of G to each vertex in the i^{th} copy of H is known as the corona product GoH of two graphs G and H (*cf.* [11, 13]).

Definition 1.2. The rooted product graph of two graphs G and H is produced by taking $|V(G)|$ copies of H, and associate v_i with the root node of i^{th} copy of H for each vertex $v_i \in G$ (*cf.* [6]).

Definition 1.3. A graph is a single-edge connected graph if the removal of one edge leads to the disconnection of the graph (*cf.* [6]).

The EV degree of an edge & VE degree of a vertex are denoted by $\delta^{ev}(e)$ and $\delta^{ve}(u)$ and are defined as follows (*cf.* [5, 10]):

The cardinality of a vertex set of the union of closed neighborhoods of the vertices u and v.

The cardinality of edges which is incident to any vertex of the closed neighborhood of u.

Let H_1 and H_2 are cycle graphs with number of vertices n and m respectively i.e., $|V(H_1)| = n$ and $|V(H_2)| = m$.

The formulae of EV-degree and VE-degree dependent variants of some topological indices are included in Table 1 (*cf.* [7, 10, 16]).

Topological Index	Expression
EV-degree Zagreb Index	$M^{ev}(H) = \sum_{e\in E(H)} d^{ev}(e)^2$
EV-degree Randic Index	$R^{ev}(H) = \sum_{e\in E(H)} d^{ev}(e)^{-\frac{1}{2}}$
EV-degree Modified Zagreb Index	${}^{m}M^{ev}(H) = \sum_{e\in E(H)} \frac{1}{d^{ev}(e)^2}$
EV-degree Forgotten Index	$F^{ev}(H) = \sum_{e\in E(H)} d^{ev}(e)^3$
VE-degree First Zagreb Alpha Index	$M_1^{\alpha ve}(H) = \sum_{v\in V(H)} d^{ve}(v)^2$
VE-degree First Zagreb Beta Index	$M_1^{\beta ve}(H) = \sum_{uv\in E(H)} \left[d^{ve}(u) + d^{ve}(v)\right]$
VE-degree Second Zagreb Index	$M_2^{ve}(H) = \sum_{uv\in E(H)} \left[d^{ve}(u)d^{ve}(v)\right]$
VE-degree Randic Index	$R^{ve}(H) = \sum_{uv\in E(H)} \left[d^{ve}(u)d^{ve}(v)\right]^{-\frac{1}{2}}$
VE-degree Harmonic Index	$H^{ve}(H) = \sum_{uv\in E(H)} \frac{2}{d^{ve}(u) + d^{ve}(v)}$
VE-degree Sum Connectivity Index	$\chi^{ve}(H) = \sum_{uv\in E(H)} \left[d^{ve}(u) + d^{ve}(v)\right]^{-\frac{1}{2}}$
VE-degree Geometric-Arithmetic Index	$GA^{ve}(H) = \sum_{uv\in E(H)} \frac{2\sqrt{d^{ve}(u)d^{ve}(v)}}{da^{ve}(u) + d^{ve}(v)}$
VE-degree ABC Index	$ABC^{ve}(H) = \sum_{uv\in E(H)} \sqrt{\frac{d^{ve}(u) + d^{ve}(v) - 2}{d^{ve}(u)d^{ve}(v)}}$
VE-degree Modified Zagreb Index	${}^{m}M^{ve}(H) = \sum_{v\in V(H)} \frac{1}{d^{ve}(v)^2}$
VE-degree Forgotten Index	$F^{ve}(H) = \sum_{v\in V(H)} d^{ve}(v)^3$

Table 1: Toplogical indices of graph H based on δ^{ev} and δ^{ve}

1.2. Methodology

We obtained the results by using analytical method, edge partition method, graph theoretical method, degree sum of neighbors method. Also we used MATLAB and MAPLE 2015 for graphical representations.

2. Main results

2.1. Corona product of two cycle graphs

Let τ be the corona product of cycle graphs H_1 and H_2 having $2mn+n$ cardinality of edges and $mn+n$ cardinality of vertices.

We observe that the edge partition of VE-degree and EV-degree of vertices and edges of τ are tabulated as follows:

EV-degree	Cardinality of edges
$2m+4$	n
5	mn
$3+m$	mn

Table 2: Edge partition of τ based on δ^{ev}

VE-degree	Cardinality of vertices
$4+4m$	n
$6+m$	mn

Table 3: Vertex partition of τ based on δ^{ve}

VE-degree	Cardinality of end vertices
$(4+4m, 4+4m)$	n
$(6+m, 6+m)$	mn
$(4+4m, 6+m)$	mn

Table 4: Edge partition of end vertices $uv \in \tau$ based on δ^{ve}

Theorem 2.1. *For the corona product τ of two cycle graphs H_1 and H_2 having $2mn+n$ cardinality of edges and $mn+n$ cardinality of vertices, EV-degree based topological indices are given by,*

1) *EV-degree Zagreb Index, $M^{ev}(\tau) = m^3n + 10m^2n + 50mn + 16n$.*

2) *EV-degree modified Zagreb Index, ${}^mM^{ev}(\tau) = \dfrac{n}{(2m+4)^2} + \dfrac{mn}{25} + \dfrac{mn}{(m+3)^2}$.*

3) *EV-degree Randic Index, $R^{ev}(\tau) = \dfrac{n}{\sqrt{2m+4}} + \dfrac{mn}{\sqrt{5}} + \dfrac{mn}{\sqrt{m+3}}$.*

4) *EV-degree Forgotten Index, $F^{ev}(\tau) = m^4n + 17m^3n + 75m^2n + 248mn + 64n$.*

Proof. From Table 2, edge partition of EV-degree of edges of τ, we compute EV-degree based topological indices:

1) We have

$$\begin{aligned} M^{ev}(\tau) &= \sum_{e\in E(\tau)} \delta^{ev}(e)^2 \\ &= n(2m+4)^2 + mn(5)^2 + mn(m+3)^2 \\ &= n(4m^2+16m+16) + 25mn + mn(m^2+6m+9) \\ &= m^3n + 10m^2n + 50mn + 16n. \end{aligned}$$

2) We have

$$\begin{aligned}
{}^{m}M^{ev}(\tau) &= \sum_{e\in E(\tau)} \frac{1}{\delta^{ev}(e)^2} \\
&= n\frac{1}{(2m+4)^2} + \frac{mn}{25} + \frac{mn}{(m+3)^2} \\
&= \frac{n}{(2m+4)^2} + \frac{mn}{25} + \frac{mn}{(m+3)^2}.
\end{aligned}$$

3) We have

$$\begin{aligned}
R^{ev}(\tau) &= \sum_{e\in E(\tau)} \delta^{ev}(e)^{-\frac{1}{2}} \\
&= n(2m+4)^{-\frac{1}{2}} + \frac{mn}{\sqrt{5}} + \frac{mn}{\sqrt{m+3}} \\
&= \frac{n}{\sqrt{2m+4}} + \frac{mn}{\sqrt{5}} + \frac{mn}{\sqrt{m+3}}.
\end{aligned}$$

4) We have

$$\begin{aligned}
F^{ev}(\tau) &= \sum_{e\in E(\tau)} \delta^{ev}(e)^3 \\
&= n(2m+4)^3 + mn(5)^3 + mn(m+3)^3 \\
&= 8m^3n + 48m^2n + 96mn + 64n + 125mn + m^4n + 9m^3n + 27m^2m + 27mn \\
&= m^4n + 17m^3n + 75m^2n + 248mn + 64n.
\end{aligned}$$

□

Theorem 2.2. *For the corona product τ of two cycle graphs H_1 and H_2 having $2mn+n$ cardinality of edges and $mn+n$ cardinality of vertices, VE-degree based topological indices are given by,*

1) *VE-degree First Zagreb Alpha Index, $M_1^{\alpha ve}(\tau) = m^3n + 28m^2n + 68mn + 16n$.*

2) *VE-degree First Zagreb Beta Index, $M_1^{\beta ve}(\tau) = 7m^2n + 30mn + 8n$.*

3) *VE-degree Second Zagreb Index, $M_2^{ve}(\tau) = 5m^3n + 56m^2n + 92mn + 16n$.*

4) *VE-degree Randic Index, $R^{ve}(\tau) = \dfrac{n}{4m+4} + \dfrac{mn}{m+6} + \dfrac{mn}{\sqrt{(4m+4)(m+6)}}$.*

5) *VE-degree Harmonic Index, $H^{ve}(\tau) = \dfrac{n}{4m+4} + \dfrac{mn}{m+6} + \dfrac{2mn}{5m+10}$.*

6) *VE-degree Sum connectivity Index, $\chi^{ve}(\tau) = \dfrac{n}{\sqrt{8m+8}} + \dfrac{mn}{\sqrt{2m+12}} + \dfrac{mn}{\sqrt{5m+10}}$.*

7) *VE-degree ABC Index, $ABC^{ve}(\tau) = n\sqrt{\frac{8m+6}{(4m+4)^2}} + mn\sqrt{\dfrac{2m+10}{(m+6)^2}} + mn\sqrt{\dfrac{5m+8}{(4m+4)(m+6)}}$.*

8) *VE-degree Geometric-Arithmetic Index, $GA^{ve}(\tau) = n(m+1) + \dfrac{4mn\sqrt{m^2+7m+6}}{5(m+2)}$.*

9) *VE-degree Modified Zagreb Index, ${}^{m}M^{ve}(\tau) = \dfrac{4m^2n + 5mn + 6n}{4(m+1)(m+6)}$.*

10) *VE-degree Forgotten Index,* $F^{ve}(\tau) = m^4n + 82m^3n + 300m^2n + 408mn + 64n.$

Proof. From Table 3 and Table 4, edge partition of VE-degree of edges of τ , we compute the following VE-degree based topological indices:

1) $M_1^{\alpha ve}(\tau) = \sum_{v \in V(\tau)} \delta^{ve}(v)^2$. From Table 3, we have

$$\begin{aligned} M_1^{\alpha ve}(\tau) &= n(4m+4)^2 + mn(m+6)^2 \\ &= 16m^2n + 32mn + 16n + m^3n + 12m^2n + 36mn \\ &= m^3n + 28m^2n + 68mn + 16n. \end{aligned}$$

2) $M_1^{\beta ve}(\tau) = \sum_{uv \in E(\tau)} [\delta^{ve}(u) + \delta^{ve}(v)]$. From Table 4, we have

$$\begin{aligned} M_1^{\beta ve}(\tau) &= n[(4m+4) + (4m+4)] + mn[(m+6) + (m+6)] + mn[(4m+4) + (m+6)] \\ &= 8mn + 8n + 2m^2n + 12mn + 5m^n + 10mn \\ &= 7m^2n + 30mn + 8n. \end{aligned}$$

3) $M_2^{ve}(\tau) = \sum_{uv \in E(\tau)} [\delta^{ve}(u)\delta^{ve}(v)]$. From Table 4, we have

$$\begin{aligned} M_2^{ve}(\tau) &= n(4m+4)^2 + mn(m+6)^2 + mn(4m+4)(m+6) \\ &= 16m^2n + 32mn + 16n + m^n + 12m^n + 36mn + 4m^3n + 24m^2n + 4m^2n + 24mn \\ &= 5m^3n + 56m^2n + 92mn + 16n. \end{aligned}$$

4) $R^{ve}(\tau) = \sum_{uv \in E(\tau)} [\delta^{ve}(u)\delta^{ve}(v)]^{-\frac{1}{2}}$. From Table 4, we have

$$\begin{aligned} R^{ve}(\tau) &= n[(4m+4)(m+6)]^{-\frac{1}{2}} + mn[(m+6)(m+6)]^{-\frac{1}{2}} + mn[(4m+4)(m+6)]^{-\frac{1}{2}} \\ &= \frac{n}{4m+4} + \frac{mn}{m+6} + \frac{mn}{\sqrt{(4m+4)(m+6)}}. \end{aligned}$$

5) $H^{ve}(\tau) = \sum_{uv \in E(\tau)} \frac{2}{\delta^{ve}(u) + \delta^{ve}(v)}$. From Table 4, we have

$$\begin{aligned} H^{ve}(\tau) &= \frac{2n}{8m+8} + \frac{2mn}{2m+12} + \frac{2mn}{5m+10} \\ &= \frac{n}{4m+4} + \frac{mn}{m+6} + \frac{2mn}{5m+10}. \end{aligned}$$

6) $\chi^{ve}(\tau) = \sum_{uv \in E(\tau)} [\delta^{ve}(u) + \delta^{ve}(v)]^{-\frac{1}{2}}$. From Table 4, we have

$$\begin{aligned} \chi^{ve}(\tau) &= n(8m+8)^{-\frac{1}{2}} + mn(2m+12)^{-\frac{1}{2}} + mn(5m+10)^{-\frac{1}{2}} \\ &= \frac{n}{\sqrt{8m+8}} + \frac{mn}{\sqrt{2m+12}} + \frac{mn}{\sqrt{5m+10}}. \end{aligned}$$

7) $ABC^{ve}(\tau) = \sum_{uv\in E(H)} \sqrt{\frac{\delta^{ve}(u)+\delta^{ve}(v)-2}{\delta^{ve}(u)\delta^{ve}(v)}}$. From Table 4, we have

$$ABC^{ve}(\tau) = n\sqrt{\frac{8m+6}{(4m+4)^2}} + mn\sqrt{\frac{2m+10}{(m+6)^2}} + mn\sqrt{\frac{5m+8}{(4m+4)(m+6)}}.$$

8) $GA^{ve}(\tau) = \sum_{uv\in E(\tau)} \frac{2\sqrt{\delta^{ve}(u)\delta^{ve}(v)}}{\delta^{ve}(u)+\delta^{ve}(v)}$. From Table 4, we have

$$GA^{ve}(\tau) = 2n\,\frac{4m+4}{8m+8} + 2mn\,\frac{m+6}{2m+12} + 2mn\,\frac{2\sqrt{(m+1)(m+6)}}{5m+10}.$$

After simplification we get,

$$GA^{ve}(\tau) = n(m+1) + \frac{4mn\sqrt{m^2+7m+6}}{5(m+2)}.$$

9) ${}^{m}M^{ve}(\tau) = \sum_{v\in V(\tau)} \frac{1}{\delta^{ve}(v)^2}$ From Table 3,

$$\begin{aligned} {}^{m}M^{ve}(\tau) &= n\,\frac{1}{4m+4} + mn\,\frac{1}{m+6} \\ &= \frac{4m^2n+5mn+6n}{4(m+1)(m+6)}. \end{aligned}$$

10) $F^{ve}(\tau) = \sum_{v\in V(\tau)} \delta^{ve}(v)^3$. From Table 3,

$$\begin{aligned} F^{ve}(\tau) &= n(4m+4)^3 + mn(m+6)^3 \\ &= 64m^3n + 192m^2n + 192mn + 64n + m^4n + 18m^3n + 108m^2n + 216mn \\ &= m^4n + 82m^3n + 300m^2n + 408mn + 64n. \end{aligned}$$

□

2.2. *Rooted product graph of two cycle graphs*

Let ξ be the rooted product graph of two cycle graphs H_1 and H_2 having $mn+n$ number of edges and mn number of vertices.

We observe that the edge partition of VE-degree and EV-degree of vertices and edges of ξ are tabulated as follows:

EV-degree	Cardinality of edges
8	n
6	$2n$
4	$(m-2)n$

Table 5: Edge partition of ξ based on δ^{ev}

VE-degree	Cardinality of vertices
12	n
6	$2n$
4	$(m-3)n$

Table 6: Vertex partition of ξ based on δ^{ve}

VE-degree	Cardinality of end vertices
$(10, 10)$	n
$(10, 6)$	$2n$
$(6, 4)$	$2n$
$(4, 4)$	$n(m-4)$

Table 7: Edge partition of end vertices $uv \in \xi$ based on δ^{ve}

Theorem 2.3. *For the rooted product graph ξ of two cycle graphs H_1 and H_2 having $mn + n$ edges and mn vertices, EV-degree based topological indices are given by,*

1) *EV-degree Zagreb Index,* $M^{ev}(\xi) = 8n(2m + 13)$.
2) *EV-degree modified Zagreb Index,* ${}^{m}M^{ev}(\xi) = \dfrac{mn}{16} - \dfrac{31n}{576}$.
3) *EV-degree Randic Index,* $R^{ve}(\xi) = (\dfrac{1}{2\sqrt{2}} + \dfrac{2}{\sqrt{6}} - 1)n + \dfrac{mn}{2}$.
4) *EV-degree Forgotten Index,* $F^{ev}(\xi) = 8n(8m + 75)$.

Theorem 2.4. *For the rooted product graph ξ of two cycle graphs H_1 and H_2 having $mn + n$ edges and mn vertices, VE-degree based topological indices are given by,*

1) *VE-degree First Zagreb Alpha Index,* $M_1^{\alpha ve}(\xi) = 8n(2m + 21)$.
2) *VE-degree First Zagreb Beta Index,* $M_1^{\beta ve}(\xi) = 8n(m + 5)$.
3) *VE-degree Second Zagreb Index,* $M_2^{ve}(\xi) = 4n(4m + 51)$.
4) *VE-degree Randic Index,* $R^{ve}(\xi) = n\left(\dfrac{1}{10} + \dfrac{1}{\sqrt{5}} + \dfrac{1}{\sqrt{6}} - 1\right) + \dfrac{mn}{4}$.
5) *VE-degree Harmonic Index,* $H^{ve}(\xi) = \dfrac{mn}{4} - \dfrac{9}{20}$.
6) *VE-degree Sum connectivity Index,* $\chi^{ve}(\xi) = n\left(\dfrac{1}{\sqrt{5}} + \dfrac{1}{2} + \dfrac{2}{\sqrt{10}} - \dfrac{2}{\sqrt{2}}\right) + \dfrac{mn}{2\sqrt{2}}$.
7) *VE-degree ABC Index,* $ABC^{ve}(\xi) = n\sqrt{\dfrac{9}{10}} + n\dfrac{\sqrt{14}}{2} + \dfrac{4n}{\sqrt{5}}$.
8) *VE-degree Geometric-Arithmetic Index,* $GA^{ve}(\xi) = n\left[\dfrac{\sqrt{15}}{2} + \dfrac{4\sqrt{6}}{5} - 4 + m\right]$.
9) *VE-degree Modified Zagreb Index,* ${}^{m}M^{ve}(\xi) = \dfrac{n(3m-4)}{12}$.
10) *VE-degree Forgotten Index,* $F^{ve}(\xi) = 16n(4m + 123)$.

2.3. Single edge connected graph of two cycle graphs

Let ζ be the single edge connected graph of two cycle graphs H_1 and H_2 having $n(m + 2)$ number of edges and $n(m + 1)$ number of vertices.

We observe that the edge partition of VE-degree and EV-degree of vertices and edges of ζ are tabulated as follows:

ev-degree	Cardinality of edges
6	$2n$
5	$2n$
4	$(m-2)n$

Table 8: Edge partition of ζ based on δ^{ev}

VE-degree	Cardinality of vertices
9	n
7	n
5	$2n$
4	$(m-3)n$

Table 9: Vertex partition of ζ based on δ^{ve}

VE-degree	Cardinality of end vertices
$(9,9)$	n
$(7,5)$	2
$(5,4)$	2
$(4,4)$	$m-4$
$(9,7)$	n

Table 10: Edge partition of end vertices $uv \in \zeta$ based on δ^{ve}

Theorem 2.5. *For the single edge connected graph ζ of two cycle graphs H_1 and H_2 having $n(m+2)$ number of edges and $n(m+1)$ number of vertices, EV-degree based topological indices are given by,*

1) EV-degree Zagreb Index, $M^{ev}(\zeta) = 2n(8m+45)$.

2) EV-degree modified Zagreb Index, ${}^{m}M^{ev}(\zeta) = \dfrac{n(225m-38)}{3600}$.

3) EV-degree Randic Index, $R^{ve}(\zeta) = \left(\sqrt{\dfrac{2}{3}} + \dfrac{2}{\sqrt{5}} - 1\right)n + \dfrac{mn}{2}$.

4) EV-degree Forgotten Index, $F^{ev}(\zeta) = 2n(32m+277)$.

Theorem 2.6. *For the single edge connected graph ζ of two cycle graphs H_1 and H_2 having $n(m+2)$ number of edges and $n(m+1)$ number of vertices, VE-degree based topological indices are given by,*

1) VE-degree First Zagreb Alpha Index, $M_1^{\alpha ve}(\zeta) = 81n^2 + 16mn + n$.

2) VE-degree First Zagreb Beta Index, $M_1^{\beta ve}(\zeta) = 8m + 34n + 10$.

3) VE-degree Second Zagreb Index, $M_2^{ve}(\zeta) = 16m + 144n + 46$.

4) VE-degree Randic Index, $R^{ve}(\zeta) = \dfrac{n}{9} + \dfrac{2}{\sqrt{35}} + \dfrac{1}{\sqrt{5}} + \dfrac{(m-4)}{4} + \dfrac{n}{3\sqrt{7}}$.

5) VE-degree Harmonic Index, $H^{ve}(\zeta) = \dfrac{17n}{72} + \dfrac{m}{4} - \dfrac{2}{9}$.

6) VE-degree Sum connectivity Index, $\chi^{ve}(\zeta) = \dfrac{(4+3\sqrt{2})n}{12\sqrt{2}} + \dfrac{3+2\sqrt{3}}{3\sqrt{3}} + \dfrac{m-4}{2\sqrt{2}}$.

7) VE-degree ABC Index, $ABC^{ve}(\zeta) = n\sqrt{\frac{8}{9}} + 2\sqrt{\frac{5}{6}} + 2\sqrt{\frac{7}{9}} + (m-4)\sqrt{\frac{3}{4}} + n\sqrt{\frac{7}{8}}.$

8) VE-degree Geometric-Arithmetic Index, $GA^{ve}(\zeta) = n\left[\frac{8+3\sqrt{7}}{8}\right] + \left[\frac{3\sqrt{35}+8\sqrt{5}+9(m-4)}{9}\right].$

9) VE-degree Modified Zagreb Index, ${}^{m}M^{ve}(\zeta) = \frac{n(315m-121)}{1260}.$

10) VE-degree Forgotten Index, $F^{ve}(\zeta) = 2n(32m+565).$

Proof procedure of above theorems are similar to that of Theorem 2.1 and Theorem 2.2, by using the Tables 5-10 and the definitions of topological indices depend on EV & VE degrees given in the Table 1.

3. Discussion and Conclusion

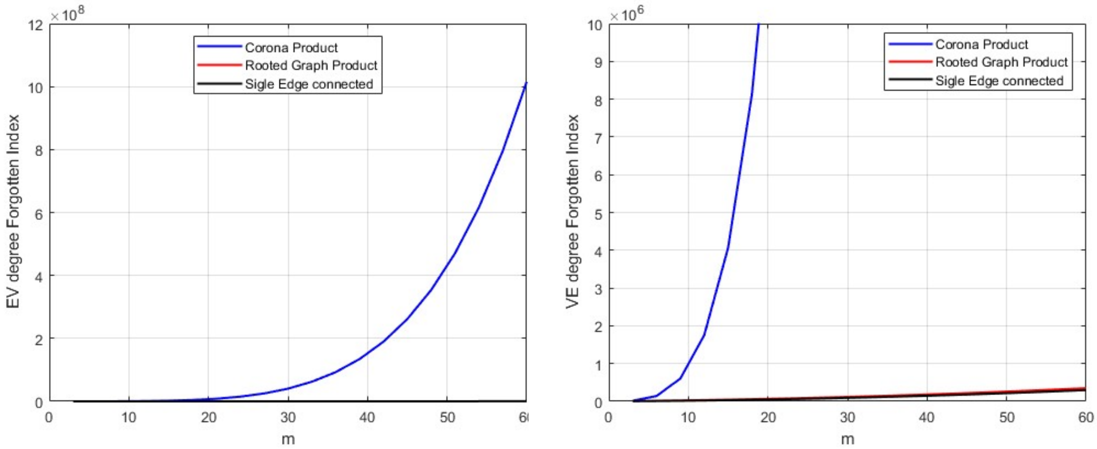
Some of the indices are graphically represented and compared the result in Figure 1 to 3. It is assumed that $m = n$ in the above comparison for simplicity.

It can be observed that all indices of rooted product and single edge connected graphs are correlated well except VE-degree Randic index. As compared to rooted product graph, m number of vertices and edges are additional in single edge connected graph, where as m^2 number of edges are additional in corona product graph.

Forgotten index of corona product graph has 4^{th} order relation with m and it is seen that value of forgotten index is very high compared to other two graphs for both VE and EV degree indices.

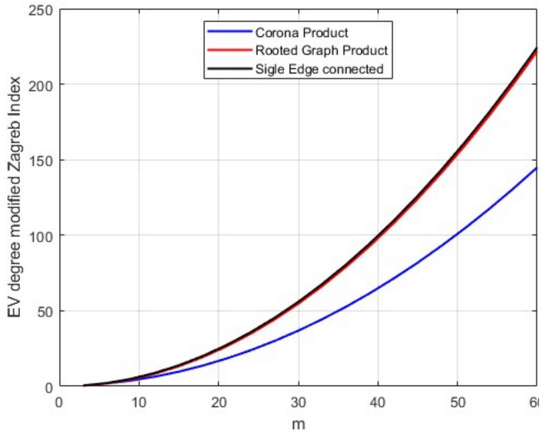
VE degree Randic index of single edge connected graph alone shows linear relation with m edges among the selected indices.

Indices derived in this paper can be used for further studies to correlate with the property of compounds, where its molecular structure is a complex arrangement of small units.

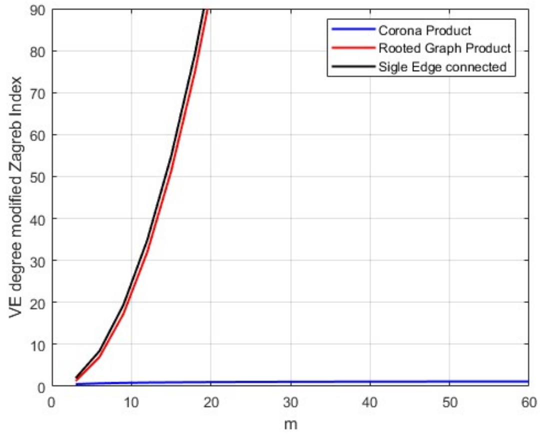


(a) Forgotten index based on δ^{ev} (b) Forgotten index based on δ^{ve}

Figure 1: EV and VE degree based forgotten indices

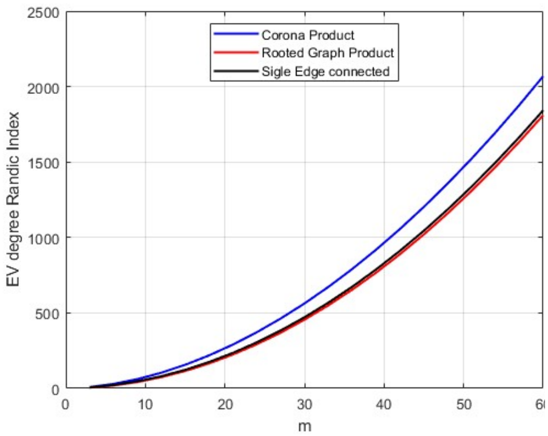


(a) Zagreb index based on δ^{ev}

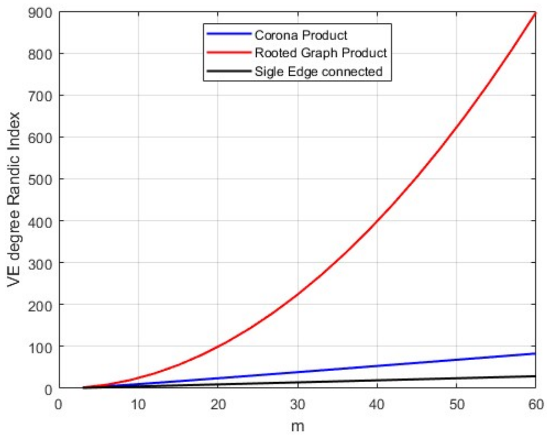


(b) Zagreb index based on δ^{ve}

Figure 2: EV and VE degree based Zagreb indices



(a) Randic index based on δ^{ev}



(b) Randic index based on δ^{ve}

Figure 3: EV and VE degree based Randic indices

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