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Revan indices and their polynomials of square snake graphs

Bhairaba Kumar Majhi [a], Veerabhadrappa Kulli [b], Ismail Naci Cangul [c]

[a] Department of Mathematics, School of Applied Sciences, Centurion University of Technology and Management, Odisha, India
[b] Department of Mathematics, Gulbarga University, Kalaburagi 585106, India
[c] Department of Mathematics, Bursa Uludag University, Gorukle 16059 Bursa, Turkey

Abstract

Molecular and spectral graph theory deals with modeling the molecular structure by a graph and to obtain some numerical value by studying this graph by mathematical methods to comment on physico-chemical properties of the molecular structure under investigation. One of the main tools to do this is Topological graph indices. There are thousands of different indices in Mathematics and Chemistry. In this research, we compute $R_1(G,x)$, $R_2(G,x)$ of square snake graphs and also $R_{01}(G)$, $R_{02}(G)$ of square snake graphs.

Keywords: Revan indices, graph, vertex degree, square snake graph

2020 MSC: 05C05, 05C12, 05C35

1. Introduction

In this research, we consider all graphs are finite, simple and connected. Let $|V| = n$ and $|E| = m$ denote the number of vertices and edges of a graph G. The distance $d(u,v)$ or $d_G(u,v)$ is the length of the shortest path between u and v in G. Let $d_G(v)$ denotes the number of vertices that are adjacent to v. Let $\Delta(G)$ denotes the maximum degree and $\delta(G)$ denotes the minimum degree of the vertices of G. For any vertex $v \in G$, $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$.

Recently, a wide range of vertices-degree-based graph invariants (Topological indices) have been introduced and extensively studied in [QSPRs/QSARs] study. The topological indices of graphs have wide area of applications establishing correlations between the structure of a molecular compound and its physic-chemical activity. The main goal of a topological graph index is to assign a numerical value to each chemical structure while keeping it as selective as possible. Various indices have been utilized to obtain necessary information about a graph molecular structure and to correlate various physico-chemical and biological aspects.

Snake graphs are planar bipartite graphs with finite or infinite one- or two-dimensional repetitions of some geometric shape. They are studied in different contexts in mathematics and other sciences. Graph Labeling is an important subject in graph theory where graphs are used to model real life situations. The Zagreb indices are the largest class of topological graph indices they have been defined and studied in [10, 11]. Some important Zagreb indices are defined and computed in [18]-[20]. The Randic index, the most well-known indices were studied in [15]. In [14], some topological indices of pentagonal chains are calculated. In [5]-[7], $R_1(G)$ and $R_2(G)$ of a graph G are defined as:

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Email addresses: bhairabamajhi@gmail.com (Bhairaba Kumar Majhi), vrkulli@gmail.com (Veerabhadrappa Kulli), cangul@uludag.edu.tr (Ismail Naci Cangul)

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⋆*Corresponding Author:* Bhairaba Kumar Majhi

$$R_1(G) = \sum_{uv\in E(G)} [r_G(u) + r_G(v)],$$

$$R_2(G) = \sum_{uv\in E(G)} [r_G(u).r_G(v)].$$

$R_1(G, x)$ and $R_2(G, x)$ are defined as

$$R_1(G, x) = \sum_{uv\in E(G)} x^{r_G(u)+r_G(v)},$$

$$R_2(G, x) = \sum_{uv\in E(G)} x^{r_G(u).r_G(v)}.$$

In [5]-[7], the first and third Revan vertex index of a graph G is defined as

$$R_{01}(G) = \sum_{u\in v(G)} r_G(u)^2.$$

$$R_3(G) = \sum_{uv\in E(G)} |r_G(u) - r_G(v)|.$$

The first and third Revan vertex polynomials are defined as

$$R_{01}(G, x) = \sum_{u\in V(G)} x^{r_G(u)^2}.$$

$$R_3(G, x) = \sum_{uv\in E(G)} x^{|r_G(u)-r_G(v)|}.$$

In [5]-[7], several other indices were studied. Also, many topological indices were studied (see [4, 9, 12]). In this research, we introduce and compute some Revan indices and their polynomials of the Square Snake graph $C^1_{4,k}$.

2. Results for square snake graphs $C^1_{4,k}$

By replacing every edge of a path P_n by a square C_4 we obtain a Square Snake Graph $C^1_{4,k}$ which is depicted in the Figure 1.

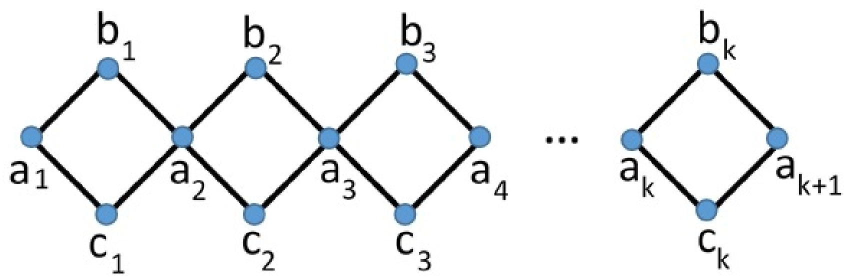

Figure 1. Square snake graph $C^1_{4,k}$

For the square snake graph $C^1_{4,k}$, we noticed $|V(G)| = 3k + 1$ and $|E(G)| = 4k$. We find that the vertex degrees are either 2 and 4 in $C^1_{4,k}$.

In Table 1, the vertex partition of $C^1_{4,k}$ is given:

d_i	$\sharp\, d_i$
2	$2k+2$
4	$k-1$

Table 1. Vertex partition of $C^1_{4,k}$

Also, in Table 2, the edge partition of $C^1_{4,k}$ is shown:

(d_i, d_j)	$\sharp\,(d_i, d_j)$
(2,2)	4
(2,4)	$4(k-1)$

Table 2. Edge partition of $C^1_{4,k}$

In the following theorem, we compute $R_{01}(C^1_{4,k})$ and $R_{01}(C^1_{4,k}, x)$ of the square snake graph $C^1_{4,k}$ by considering the above vertex and edge partitions.

Theorem 2.1. *Let G be the square snake graph $C^1_{4,k}$. Then we have the following results:*

$$\begin{aligned} R_{01}(C^1_{4,k}) &= 36k+28, \\ R_{01}(C^1_{4,k}, x) &= (2k+2)x^{16} + (k-1)x^4. \end{aligned}$$

Proof. Let G be the square snake graph $C^1_{4,k}$ and $|V(G)| = 3k+1$.Thus, we have $\Delta(G) = 4$, $\delta(G) = 2$. The vertex $V(C^1_{4,k})$ can be partitioned as,

$$V_2 = \{u \in V(G) \mid d_G(u) = 2\}, |V_2| = 2k+2,$$
$$V_4 = \{u \in V(G) \mid d_G(u) = 4\}, |V_4| = k-1.$$

Clearly, we find $\Delta(G) + \delta(G) = 6$. Thus, $r_G(u) = 6 - d_G(u)$.
Only two types of Revan vertices are found in the square snake graph $C^1_{4,k}$:

$$\begin{aligned} V_{r4} &= \{u \in V(G) \mid r_G(u) = 4\}, |V_{r4}| = 2k+2, \\ V_{r2} &= \{u \in V(G) \mid r_G(u) = 2\}, |V_{r2}| = k-1. \end{aligned}$$

1) To compute $R_{01}(C^1_{4,k})$, we have the following,

$$\begin{aligned} R_{01}(G) &= \sum_{u\in V_{r4}} r_G(u)^2 + \sum_{u\in V_{r2}} r_G(u)^2 \\ &= (2k+2)\times 4^2 + (k-1)\times 2^2 \\ &= 36k+28. \end{aligned}$$

2) To compute $R_{01}(C^1_{4,k}, x)$, we have the following,

$$\begin{aligned} R_{01}(G, x) &= \sum_{u\in V_{r4}} x^{r_G(u)^2} + \sum_{u\in V_{r2}} x^{r_G(u)^2} \\ &= (2k+2)\times x^{4^2} + (k-1)\times x^{2^2} \\ &= (2k+2)\times x^{16} + (k-1)\times x^4. \end{aligned}$$

□

We compute the values of $R_1(C^1_{4,k}),\ R_2(C^1_{4,k}),\ R_3(C^1_{4,k})$ for square snake graph network.

Theorem 2.2. *Let $C^1_{4,k}$ be the square snake graph. Then*

$$\begin{aligned} R_1(C^1_{4,k}) &= 8(3k+1), \\ R_2(C^1_{4,k}) &= 32(k+1), \\ R_3(C^1_{4,k}) &= 8(k-1). \end{aligned}$$

Proof. Let G be the square snake graph $C^1_{4,k}$ and $|E(G)| = 4k$. Then, we have the following results,

$$\begin{aligned} E_{2,2} &= \{u \in E(G) \mid d_G(u) = d_G(v) = 2\}, \ \ |E_{2,2}| = 4, \\ E_{2,4} &= \{u \in V(G) \mid d_G(u) = d_G(v) = 4\}, \ \ |E_{2,4}| = 4(k-1). \end{aligned}$$

Clearly, we have $\Delta(G) = 4, \delta(G) = 2$. Thus we have $r_G(u) = 6 - d_G(u)$. Only two types of Revan edges are found in the square snake graph $C^1_{4,k}$:

$$\begin{aligned} RE_{4,4} &= \{uv \in E(G) \mid r_G(u) = 4,\ r_G(v) = 4\}, \ \ |RE_{4,4}| = 4, \\ RE_{4,2} &= \{uv \in V(G) \mid r_G(u) = 4,\ r_G(v) = 2\}, \ \ |RE_{4,2}| = 4(k-1). \end{aligned}$$

1) To compute $R_1(C^1_{4,k})$ we have the following,

$$\begin{aligned} R_1(G) &= \sum_{uv \in E(G)} [r_G(u) + r_G(v)] \\ &= \sum_{RE_{4,4}} [r_G(u) + r_G(v)] + \sum_{RE_{4,2}} [r_G(u) + r_G(v)] \\ &= 4(4+4) + 4(k-1)(4+2) \\ &= 8(3k+1). \end{aligned}$$

2) To compute $R_2(C^1_{4,k})$ we have the following,

$$\begin{aligned} R_2(G) &= \sum_{uv \in E(G)} [r_G(u) \times r_G(v)] \\ &= \sum_{RE_{4,4}} [r_G(u) \times r_G(v)] + \sum_{RE_{4,2}} [r_G(u) \times r_G(v)] \\ &= 4(16) + 4(k-1)(8) \\ &= 32(k+1). \end{aligned}$$

3) To compute $R_3(C^1_{4,k})$ we have the following,

$$\begin{aligned} R_3(G) &= \sum_{uv \in E(G)} [r_G(u) - r_G(v)] \\ &= \sum_{RE_{4,4}} |r_G(u) - r_G(v)| + \sum_{RE_{4,2}} |r_G(u) \times r_G(v)| \\ &= 8(k-1). \end{aligned}$$

□

In the following theorem, for square snake graph networks $C^1_{4,k}$ we compute the values of $R_1(C^1_{4,k}, x),\ R_2(C^1_{4,k}, x)$, $R_3(C^1_{4,k}, x)$ for square snake graph networks.

Theorem 2.3. *Let $C^1_{4,k}$ be the square snake graph network. Then*

$$\begin{aligned} R_1(C^1_{4,k}, x) &= 4x^8 + 4(k-1)x^6, \\ R_2(C^1_{4,k}, x) &= 4x^{16} + 4(k-1)x^8, \\ R_3(C^1_{4,k}, x) &= 4(k-1)x^2. \end{aligned}$$

Proof. **1)** Using the Revan edge set partition of the square snake graph networks $C^1_{4,k}$ and applying the formula of $R_1(G, x)$ of G we have,

$$\begin{aligned} R_1(C^1_{4,k}, x) &= \sum_{RE_{4,4}} x^{r_G(u)+r_G(v)} + \sum_{RE_{4,2}} x^{r_G(u)+r_G(v)} \\ &= 4x^{4+4} + 4(k-1)x^{4+2} \\ &= 4x^8 + 4(k-1)x^6. \end{aligned}$$

2) Using the Revan edge set partition of the square snake graph networks $C^1_{4,k}$ and applying the formula of $R_2(G, x)$ of G we have,

$$\begin{aligned} R_2(C^1_{4,k}, x) &= \sum_{RE_{4,4}} x^{r_G(u)\times r_G(v)} + \sum_{RE_{4,2}} x^{r_G(u)\times r_G(v)} \\ &= 4x^{16} + 4(k-1)x^8. \end{aligned}$$

3) Using the Revan edge set partition of the square snake graph networks $C^1_{4,k}$ and applying the formula of $R_3(G, x)$ of G we have,

$$\begin{aligned} R_3(C^1_{4,k}, x) &= \sum_{RE_{4,4}} x^{[r_G(u)-r_G(v)]} + \sum_{RE_{4,2}} x^{[r_G(u)-r_G(v)]} \\ &= 4(k-1)x^2. \end{aligned}$$

□

3. Conclusion

In this research work, we considered a special graph network structures $C^1_{4,k}$ [square snake graphs] and computed the Revan indices and their polynomials.

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