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Mitigating the issue of unemployment through entrepreneurship development in MSME sector of India: A mathematical model analysis

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Abstract

Entrepreneurship ventures in Micro, Small and Medium Enterprises (MSMEs) are the crucial components of India's economic growth and development. In 2020 - 2021 MSMEs has contributed almost 30% in India's GDP. MSMEs provide an essential platform to unemployed youths to venture in various small scale business opportunities using mostly the indigenous infrastructure and production models resources. By creating own MSME company a previously job seeker becomes a job giver to the other unemployed individuals. Unemployment is another important socioeconomic issue in India. After agriculture, MSME sector of India is the largest employment generator employing almost 15 million people all over the country. Therefore, by developing adequate entrepreneurship ventures in MSME sector the unemployment issue of India can be mitigated. The main objective of this article is to assimilate the dynamics of MSMEs to resolve the challenges of unemployment by creating jobs in new MSME entrepreneurships. For that purpose, we have proposed a deterministic mathematical model using nonlinear differential equations and analyzed it analytically as well as numerically. The model parameters have been estimated using real data set. The sensitivity analysis of the parameters has also been performed using PRCC method. It has been found that the rate at which an unemployed individual becomes an MSME entrepreneur with proper skills and training, the governmental schemes to financially support the MSMEs and the rate of generation of employment in MSME sector can significantly alter the system dynamics. Moreover, during COVID - 19 pandemic the rate of at which MSMEs were closed due to financial crisis & lockdowns and the rate at which MSME employees lost their jobs are also important regulators of the issue of unemployment. The results obtained through this study can be helpful to constitute the future financial policies related to the development of MSME sector of India.

Keywords: Unemployment, entrepreneurship, MSME, mathematical model, data fitting, sensitivity analysis, numerical simulations

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1. Introduction

Entrepreneurship is the ability and willingness to develop, organize and successfully run a business enterprize, along with coping up against any of its uncertainties in order to convert the business into a profitable one. The basic aim of entrepreneurship is to create or extract of some specific economic value (*cf.* [1]). Schumpeter [11] defined an entrepreneur as a person who is willing and successful to convert a new innovative idea into a successful innovation

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or an efficient business venture (*cf.* [11]). It is the responsibility of the entrepreneur to develop the basic ideas to initiate the business. Moreover, while constructing the business, the entrepreneur has to undertake the associated financial risks, supervise the necessary resources, arrange required infrastructure and develop innovative ideas. The business that is established through the process of entrepreneurship is called an enterprize. Therefore, an entrepreneur is that individual who coordinates the process of entrepreneurship to develop an enterprize. Entrepreneurship is considered to be one of the pivotal aspects of the socioeconomic growth and development of a country, especially a developing one. In India we can see mixed economic system where state and private entrepreneurship initiatives both coexist. However, the small industrial sectors and enterprizes are mostly managed by the private entrepreneurs. To protect the interests of small-scale entrepreneurs, Government of India introduced MSMED (Micro Small Medium Enterprizes Development) Act 2006 (*cf.* [17]). This act legally classified the enterprizes into three assorted categories namely, Micro, Small, Medium Enterprizes (MSMEs). A revised criteria of classification in MSME was announced by the Ministry of MSME, Govt of India in 2020 (*cf.* [19]). MSMEs act as complementary business ventures to large, established and branded industries and form an important platform for building a conducive and supportive environment for indigenous skills, innovations and entrepreneurship development initiatives.

1.1. Role of MSME in economic development of India

India is considered one of the fastest growing economies in the world. MSMEs are an integral and highly important part of Indian economy. According to a report by Ministry of MSME, Government of India (*cf.* [16]) MSMEs have contributed almost 30% in the GDP of Indian Economy in the 2021 - 2022 fiscal year. Up to August 2022, 42.67% of India's export has been MSME based products. So far this economic sector in India has shown a stable performance and has safeguarded the Indian economy from global adversities and uncertainties. From simple to high-precision and sophisticated products, MSME sector of India produces a large variety of consumer goods. MSME segment of Indian industry in fact has been encouraged and supported by Government of India in both the pre - reform and post - reform period of economic development to fulfil the objective of self - reliant India or AATMA NIRBHAR BHARAT as well as development of rural industrialization. Indian MSME sector consists of both of "traditional" as well as "modern" small scale industries. This sector has eight basic subgroups, namely, (i) handlooms, (ii) handicrafts, (iii) coir, (iv) sericulture, (v) khadi, (vi) village industries, (vii) small scale industries and (vii) power looms (*cf.* [16]). Various small scale village industries, khadi and coir segments are major and profiting contributors to the growth and development of MSME based economic structure of India. As a result of the innovative capabilities and low - cost, highly skilled manufacturing abilities, presently various Indian MSMEs have started partnerships with foreign industrial farms for mutual economic benefit.

1.2. Generation of employment in the MSME sector of India

Like all other developing nations, unemployment is a concerning socioeconomic issue in India. Periodic labor force survey by Government of India, during January 2023 to December 2023 finds that, in rural India unemployment rate is 2.4% (*cf.* [18]). Whereas, in urban areas it is 5.4% . Youth (persons of age group 15 - 29 years) unemployment rate in 2022 - 2023 is 13.2%. Hence unemployment is still a major socioeconomic concern, especially among the youth force of India. MSME sector can play a major role in resolving this issue. According to the National Sample Survey (NSS) 73rd Round that was conducted during 2015 - 16, Indian MSME sector has generated almost 11.10 crore jobs (*cf.* [16]). 60.41 lakh in Manufacturing, 0.07 lakh in Non-captive Electricity Generation and Transmission, 387.18 lakh in Trade and 362.82 lakh in Other Services. According to Udyam Registration Portal of Government of India, till December 2022, approximately 1.28 crore registered MSME industries have provided employment for 9.31 crore people. Among them 2.18 crore are women.

1.3. Literature survey

Various researchers have studied the economical impact of MSMEs in India. The study of Das discusses the enormous growth potential and opportunities of MSME sector for economic development of India using secondary data (*cf.* [2]). He also identified the important issues, challenges, and suggestions in this regard. The performance and impacts of MSMEs in Indian economy have been studied by Lama (*cf.* [6]). She has also discussed the impacts of various governmental policies to resolve different issues associated with the growth of MSME sector of India. This study concludes that the MSMEs require to improve their productivity and quality by proper innovation and also

reduce product cost for sustainability. By analyzing the current trends of MSMEs, P. Manna and T. Mistri showed that, though micro enterprizes lead the MSME sector in India, some developed states are developing their small and medium enterprizes to decrease regional disparity (*cf.* [7]). Mohanty analyzed the present economic status of the Indian MSME sector and effect of various governmental policies for their development by resolving various issues and challenges (*cf.* [8]). Unni studied the impact of COVID - 19 on the Indian MSME sector (*cf.* [14]). He theorized that there is an urgent need to address the issue, in short term basis for the of loss of employment, revenue, and GDP for further economic growth.

Researches have also been performed on the role of MSMEs to curb the issue of unemployment through generation of employments. Using exploratory methodology and secondary data, Zanjurne showed that the MSME sector significantly contributes to employment generation (*cf.* [15]). Shelly et al. studied the employment opportunities in MSME sector through the Green Jobs, that contribute or preserve the environment, generated by indigenous MSME sectors of India (*cf.* [12]). Using secondary data for the khadi and village industry, sericulture industry and coir industry based MSMEs they showed that MSME sector significantly generates Green Jobs that puts positive impact to resolve the issue of unemployment in India. Das and Chakraborty performed a time series analysis with a statistical forecasting model for manufacturing and engineering based MSMEs of India (*cf.* [3]). Using secondary data, they have predicted possible job creations between the time 2016 to 2021. Gade performed a data-based study and showed that MSMEs are generating more employment opportunities per unit, than the other industrial sectors of India (*cf.* [4]). If this generation of employment is to be sustained, then the uniqueness of the MSMEs needs to preserved and protected in an overt and explicit manner.

1.4. Impact of COVID-19 on MSME sector of India

One of the sectors of Indian economy that has been negatively affected by COVID - 19 pandemic is MSME sector. Due to its weak financial structure, low operational size, small business scale and limited accessibility to financial resources this sector has suffered immensely. According to M. Saritha around 95% MSME firms were adversely affected due to nation-wide lockdown that started in April 2020 (*cf.* [10]). By August 2020, 70% of MSMEs got affected due to consecutive lockdowns. Around February 2021, almost 40% of MSMEs were suffering from financial crisis. Business volume of Indian MSMEs decreased almost 11% due to lockdown in 2021 compared to 46% decline in 2020 (*cf.* [13]). A detailed study about the impacts of COVID - 19 on Indian MSME can be found in the study of Kharat et al. (*cf.* [5]).

To the best of our knowledge there has not been any data - based study with the help of deterministic mathematical models to analyze the dynamics of the MSMEs in India, in the perspective to mitigate the issue of unemployment through entrepreneurship development in MSME sector. We therefore, propose and study a deterministic mathematical model with the help of nonlinear differential equations to determine the interplay between the generation of MSME entrepreneurship ventures and generation of new employment opportunities to mitigate the issue of unemployment in India. What should be the rate of development of MSME entrepreneurship ventures so that the issue of unemployment can be curbed is the main objective of this study.

2. Mathematical model

First we define the system variables. Let, $U(t)$ denote the number of unemployed individuals at any time t. The density of non-branded MSME entrepreneurship companies be $E(t)$. Whereas, $C(t)$ be the same for the existing established and branded companies that enter the territory of the MSME companies. From now on we denote such established and branded companies as EB companies. In fact, when an MSME entrepreneur doing good business on some product, it is often observed that such EB companies encroach its territory to grab the market. The number of jobs created by the MSME companies are denoted by $J_E(t)$ and the same for the EB companies is specified by $J_C(t)$. The number of the individuals employed in MSME companies and EB companies are respectively represented by $S_E(t)$ and $S_C(t)$.

Further we define Δ to be the constant input of unemployed individuals in the system. The rate at which the unemployed individuals become entrepreneurs is represented by the parameter α. This rate depends on the training an unemployed individual obtains through training programs or skill development programs, to start an MSME entrepreneurship. Unemployed individuals become employed in MSMEs at a rate β_1 and in EB companies at a rate β_2.

The MSMEs compete with each other at a rate γ_1 and the same for EB companies is γ_2. The governmental schemes to support and protect the interests of MSMEs is denoted by $\theta \in [0, 1]$. Here $\theta = 0$ implies no support, whereas, $\theta = 1$ signifies 100% support. The effect of competition of EB companies on MSMEs is denoted by the parameter a. The same of MSMEs on EB companies is given by b. New jobs are created through MSMEs in a rate ϕ and at EB companies in a rate ψ. Individuals lose their jobs in MSME and EB companies at a rate κ_1 and κ_2 respectively. An employee of MSME moves to a EB company at a rate δ for better opportunity. And an EB company employee leaves his/her job and opts for a MSME company job at a rate η. The established companies encroach the business territory of MSMEs at a rate Γ. The natural death rate of U_t, $S_E(t)$, $S_C(t)$ class individuals are denoted by μ_1, μ_4, μ_5 respectively. μ_2 and μ_7 are respectively the rates at which MSME and EB companies are shut down or leaves the designated economic area under consideration. The jobs created by MSMEs and EB companies are obliterated due to financial reasons or sustainability issues at the rate μ_3 and μ_6 respectively. Moreover, we consider the following assumptions:

Assumption 1: The new and unbranded MSME entrepreneurship ventures do not have any brand value at present.

Assumption 2: Each entrepreneur individual is equivalent with a MSME company. It is possible that two or more people may start a MSME company, but we do not consider those cases, to avoid complexity.

Assumption 3: We consider that the rate of creation of new jobs by MSME ventures and EB companies does not depend on the existing number of unemployed individuals. As these companies create vacancies according to their own requirement and need. Therefore, the news jobs are created depending on the requirement of the existing number of MSMEs and EB companies.

Assumption 4: All parameters are considered to be positive.

Figure 1 represents the schema diagram of the interactions between the system variables based on the above discussion.

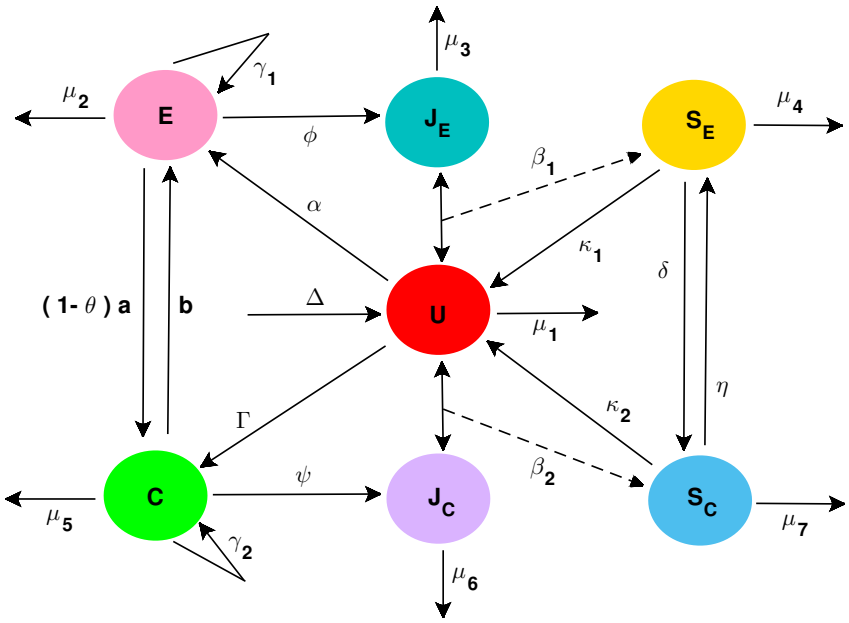

Figure 1. Schema diagram depicting the interactions among the system variables

Based on this schema diagram given in Figure 1 we propose the following deterministic mathematical model with the help of nonlinear differential equations to model the interactions between the system variables.

We study this model system (2.1):

$$\begin{aligned}
\frac{dU}{dt} &= \Delta - \alpha U - \beta_1 U J_E - \beta_2 U J_C + \kappa_1 S_E + \kappa_2 S_C - \mu_1 U,\\
\frac{dE}{dt} &= \alpha U - \gamma_1 E^2 - (1-\theta) aEC - \mu_2 E,\\
\frac{dJ_E}{dt} &= \phi E - \mu_3 J_E,\\
\frac{dS_E}{dt} &= \beta_1 U J_E - \kappa_1 S_E - \delta S_E + \eta S_C - \mu_4 S_E,\\
\frac{dC}{dt} &= \Gamma - \gamma_2 C^2 - bEC - \mu_5 C,\\
\frac{dJ_C}{dt} &= \psi C - \mu_6 J_C,\\
\frac{dS_C}{dt} &= \beta_2 U J_C - \kappa_2 S_C + \delta S_E - \eta S_C - \mu_7 S_C,
\end{aligned} \tag{2.1}$$

with respect to the following initial conditions:

$$U(0) > 0,\ E(0) \geq 0,\ J_E(0) \geq 0,\ S_E(0) \geq 0,\ C(0) > 0,\ J_C(0) \geq 0,\ S_C(0) \geq 0. \tag{2.2}$$

3. Analytical results

In this section we derive different analytical results such as positivity and boundedness of the solutions of the model system (2.1) along with existence and stability of the equilibrium points.

3.1. Positivity and boundedness

Proposition 3.1. *The solutions of the model system (2.1) are positively invariant and ultimately bounded in* $\Omega = \left\{ (U,\ E,\ J_E,\ S_E,\ C,\ J_C,\ S_C) \in \mathbb{R}^7_+ \;\middle|\; 0 < U + E + S_E + C + S_C \leq \frac{\Delta+\Gamma}{\mu}; J_E \leq \frac{\phi(\Delta+\Gamma)}{\mu\mu_3}; J_C \leq \frac{\psi(\Delta+\Gamma)}{\mu\mu_6} \right\}$, *where* $\mu = \min\{\mu_1, \mu_2, \mu_4, \mu_5, \mu_7\}$.

Proof. First we show that all the solutions of the system (2.1) starting with initial conditions (2.2) are positive, using a lemma proposed by Nagumo [9].

Lemma 3.2. *Consider a system* $\dot{X} = F(X)$ *where* $F(X) = [F_1(X), F_2(X), \ldots, F_n(X)]$, $X \in \mathbb{R}^n$, *with initial condition* $X(0) = X_0 \in \mathbb{R}^n$. *If for* $X_i = 0,\ i = 1, 2, \ldots, n$ *we get* $F_i(X)|_{X_i=0} \geq 0$, *then any solution of* $\dot{X} = F(X)$ *with given initial condition, say,* $X(t) = X(t; X_0)$ *will be positive i.e.,* $X(t) \in \mathbb{R}^n_+$.

It can be easily seen that when $U = E = J_E = S_E = C = J_C = S_C = 0$, then $\frac{dU}{dt} = \Delta > 0$, $\frac{dC}{dt} = \Gamma > 0$ and $\frac{dE}{dt} = \frac{dJ_E}{dt} = \frac{dS_E}{dt} = \frac{dJ_C}{dt} = \frac{dS_C}{dt} = 0$. Hence following Lemma 3.2 all the solutions of (2.1) starting with initial conditions (2.2) are positive.

Next we establish the boundedness of the solutions of (2.1) for all large t. Define $W(t) = U(t) + E(t) + S_E(t) + C(t) + S_C(t)$. Differentiating $W(t)$ along the solutions of (2.1) we obtain,

$$\begin{aligned}
&\dot{W}(t) = \dot{U}(t) + \dot{E}(t) + \dot{S}_E(t) + \dot{C}(t) + \dot{S}_C(t)\\
\Rightarrow &\dot{W}(t) \leq (\Delta + \Gamma) - \mu W\\
\Rightarrow &\limsup_{t\to\infty} W(t) \leq \frac{\Delta+\Gamma}{\mu},
\end{aligned} \tag{3.1}$$

where $\mu = \min\{\mu_1, \mu_2, \mu_4, \mu_5, \mu_7\}$. Therefore, for all large t, solutions of system (2.1) are uniformly bounded. Again from the third & fourth equation of (2.1) and following (3.1) one can easily derive that,

$$\begin{aligned}
\dot{J}_E(t) \leq \frac{\phi(\Delta+\Gamma)}{\mu} - \mu_3 J_E &\Rightarrow \limsup_{t\to\infty} J_E(t) \leq \frac{\phi(\Delta+\Gamma)}{\mu\mu_3},\\
\dot{J}_C(t) \leq \frac{\psi(\Delta+\Gamma)}{\mu} - \mu_6 J_C &\Rightarrow \limsup_{t\to\infty} J_C(t) \leq \frac{\psi(\Delta+\Gamma)}{\mu\mu_6}.
\end{aligned}$$

Hence the proposition is proved. □

3.2. Equilibrium points

Model system (2.1) has two equilibrium points. One is the unemployment equilibrium denoted by

$$\bar{P}(\bar{U}, 0, 0, 0, \bar{C}, 0, 0)$$

where $\bar{U} = \frac{\Delta}{\alpha+\mu_1}$ and $\bar{C} = \frac{\mu_5+\sqrt{\mu_5^2+4\gamma_2\Gamma}}{2\gamma_2}$. This equilibrium always exists. The other is the MSME equilibrium represented by $P^*(U^*, E^*, J_E^*, S_E^*, C^*, J_C^*, S_C^*)$ where,

$$\begin{aligned}
U^* &= \frac{A_{43}C^{*4} + A_{44}C^{*3} + A_{45}C^{*2} + A_{46}C^* + A_{47}}{C^{*2}},\\
S_E^* &= \frac{A_{30}C^{*6} + A_{31}C^{*5} + A_{32}C^{*4} + A_{33}C^{*3} + A_{34}C^{*2} + A_{35}C^* + A_{36}}{C^{*3}},\\
S_C^* &= \frac{A_{27}(A_{20}C^{*6} + A_{21}C^{*5} + A_{22}C^{*4} + A_{23}C^{*3} + A_{24}C^{*2} + A_{25}C^* + A_{26})}{C^{*3}},\\
E^* &= \frac{A_1 + A_2C^{*2} + A_3C^*}{C^*},\\
J_C^* &= \frac{\psi C^*}{\mu_6},\ J_E^* = \frac{\phi E^*}{\mu_3}
\end{aligned} \tag{3.2}$$

and C^* is given by the positive root of the equation:

$$B_1C^{*6} + B_2C^{*5} + B_3C^{*4} + B_4C^{*3} + B_5C^{*2} + B_6C^* + B_7 = 0.$$

Here, $A_1 = \frac{\Gamma}{b}$, $A_2 = -\frac{\gamma_2}{b}$, $A_3 = -\frac{\mu_5}{b}$, $A_4 = \frac{\beta_2\gamma_1\psi}{\alpha\mu_6(\kappa_2+\eta+\mu_7)}$, $A_5 = \frac{\beta_2(1-\theta)a\psi}{\alpha\mu_6(\kappa_2+\eta+\mu_7)}$, $A_6 = \frac{\beta_2\mu_2\psi}{\alpha\mu_6(\kappa_2+\eta+\mu_7)}$, $A_7 = A_2^2A_4 + A_2A_5$, $A_8 = 2A_2A_3A_4 + A_2A_6 + A_5A_3$, $A_9 = A_4A_3^2 + 2A_1A_2A_4 + A_1A_5 + A_3A_6$, $A_{10} = 2A_1A_3A_4 + A_1A_6$, $A_{11} = A_1^2A_4$, $A_{12} = \frac{\delta\beta_1}{(\kappa_2+\eta+\mu_7)(\kappa_1+\delta+\mu_4)}$, $A_{13} = \frac{\gamma_1\phi A_1^3}{\alpha\mu_3}$, $A_{14} = \frac{A_2^3\gamma_1\phi + A_2^2(1-\theta)a\phi}{\alpha\mu_3}$, $A_{15} = \frac{A_3^3\gamma_1\phi + 6A_1A_2A_3\gamma_1\phi + 2A_1A_3(1-\theta)a\phi + A_3^2\mu_2\phi + 2A_1A_2\mu_2\phi}{\alpha\mu_3}$, $A_{16} = \frac{3A_1^2A_2\gamma_1\phi + 3A_1A_3^2\gamma_1\phi + (1-\theta)a\phi A_1^2 + 2A_1A_3\mu_2\phi}{\alpha\mu_3}$, $A_{17} = \frac{3A_1^2A_3\gamma_1\phi + \mu_2\phi A_1^2}{\alpha\mu_3}$, $A_{18} = \frac{3A_1A_2^2\gamma_1\phi + 3A_2A_3^2\gamma_1\phi + (A_3^2+2A_1A_2)(1-\theta)a\phi + 2A_2A_3\mu_2\phi}{\alpha\mu_3}$, $A_{19} = \frac{3A_2^2A_3\gamma_1\phi + 2A_2A_3(1-\theta)a\phi + A_2^2\mu_2\phi}{\alpha\mu_3}$, $A_{20} = A_7 + A_{12}A_{14}$, $A_{21} = A_8 + A_{12}A_{19}$, $A_{22} = A_9 + A_{12}A_{18}$, $A_{23} = A_{10} + A_{12}A_{15}$, $A_{24} = A_{11} + A_{12}A_{16}$, $A_{25} = A_{12}A_{17}$, $A_{26} = A_{12}A_{13}$, $A_{27} = \frac{(\kappa_2+\eta+\mu_7)(\kappa_1+\delta+\mu_4)}{(\kappa_2+\eta+\mu_7)(\kappa_1+\delta+\mu_4)-\delta\eta}$, $A_{28} = \frac{\beta_1}{\kappa_1+\delta+\mu_4}$, $A_{29} = \frac{\eta A_{27}}{\kappa_1+\delta+\mu_4}$, $A_{30} = A_{28}A_{14} + A_{29}A_{20}$, $A_{31} = A_{28}A_{19} + A_{29}A_{21}$, $A_{32} = A_{28}A_{18} + A_{29}A_{22}$, $A_{33} = A_{28}A_{15} + A_{29}A_{23}$, $A_{34} = A_{28}A_{16} + A_{29}A_{24}$, $A_{35} = A_{28}A_{17} + A_{29}A_{25}$, $A_{36} = A_{28}A_{13} + A_{29}A_{26}$, $A_{37} = \frac{\beta_2}{\kappa_2+\eta+\mu_7}$, $A_{38} = \frac{A_7}{A_{37}}$, $A_{39} = \frac{A_8}{A_{37}}$, $A_{40} = \frac{A_9}{A_{37}}$, $A_{41} = \frac{A_{10}}{A_{37}}$, $A_{42} = \frac{A_{11}}{A_{37}}$, $A_{43} = \frac{A_2^2\gamma_1 + (1-\theta)aA_2}{\alpha}$, $A_{44} = \frac{(1-\theta)aA_3 + A_2\mu_2 + 2A_2A_3\gamma_1}{\alpha}$, $A_{45} = \frac{A_3^2\gamma_1 + 2A_1A_2\gamma_1 + (1-\theta)aA_1 + \mu_2A_3}{\alpha}$, $A_{46} = \frac{2A_1A_3\gamma_1 + A_1\mu_2}{\alpha}$, $A_{47} = \frac{\gamma_1A_1^2}{\alpha}$, $B_1 = \kappa_2A_{27}A_{20} + \kappa_1A_{30} - \beta_2A_{18} - \beta_1A_{14}$, $B_2 = \kappa_2A_{27}A_{21} + \kappa_1A_{31} - \beta_2A_{39} - \beta_1A_{19} - (\alpha+\mu_1)A_{43}$, $B_3 = \kappa_2A_{27}A_{22} + \kappa_1A_{32} - \beta_2A_{40} - \beta_1A_{18} - (\alpha+\mu_1)A_{44}$, $B_4 = \kappa_2A_{27}A_{23} + \kappa_1A_{33} - \beta_2A_{41} - \beta_1A_{15} - (\alpha+\mu_1)A_{45} + \Delta$, $B_5 = \kappa_2A_{27}A_{24} + \kappa_1A_{34} - \beta_2A_{42} - \beta_1A_{16} - (\alpha+\mu_1)A_{46}$, $B_6 = \kappa_2A_{27}A_{25} + \kappa_1A_{35} - \beta_1A_{13}$, $B_7 = \kappa_2A_{27}A_{26} + \kappa_1A_{36} - (\alpha+\mu_1)A_{47} - \beta_1A_{17}$.

Model system (2.1) will have at least one feasible P^* if the system parameters satisfy the following conditions:

$$\begin{aligned}
&\textbf{(I)}\quad \Gamma - (\gamma_2C^{*2} + \mu_5C^*) > 0,\\
&\textbf{(II)}\quad B_1B_7 < 0.
\end{aligned} \tag{3.3}$$

3.3. Stability of the equilibrium points

Proposition 3.3. *The unemployment equilibrium $\bar{P}$ is locally asymptotically stable if and only if the conditions specified in* (3.5) *are satisfied.*

Proof. The characteristic equation of the jacobian matrix associated with model system (2.1) evaluated at $\bar{P}$ takes the form

$$\xi^7 + Q_1\xi^6 + Q_2\xi^5 + Q_3\xi^4 + Q_4\xi^3 + Q_5\xi^2 + Q_6\xi + Q_7 = 0, \tag{3.4}$$

where, $Q_1 = H_{14}$, $Q_2 = H_{15}$, $Q_3 = H_{16} + F_{11}$, $Q_4 = H_{17} + G_9 + F_{10}$, $Q_5 = H_{19} + G_8 + F_9$, $Q_6 = H_{19} + G_7 + F_8$, $Q_7 = H_{20} + G_6 + F_7$, $F_1 = m_{21}m_{32}m_{77}$, $F_2 = m_{21}m_{32}$, $F_3 = m_{55}m_{66}$, $F_4 = m_{55} + m_{66}$, $F_5 = m_{13}m_{44} - m_{14}m_{43}$, $F_6 = m_{13}$, $F_7 = F_1F_2F_5$, $F_8 = -F_1F_2F_6 - F_1F_3F_6 - F_2F_3F_5$, $F_9 = -F_1F_3F_6 + F_1F_5 + F_2F_3F_6 + F_2F_4F_5$, $F_{10} = -F_1F_6 - F_2F_4F_6 - F_2F_5$, $F_{11} = F_2F_6$, $H_1 = m_{22}m_{66}$, $H_2 = m_{22}m_{66}$, $H_3 = -m_{22}m_{55}$, $H_4 = m_{22} + m_{55}$, $H_5 = m_{44}m_{77}$, $H_6 = m_{44}+m_{77}$, $H_7 = H_1H_3$, $H_8 = -H_1H_3+H_1+H_4$, $H_9 = H_3-H_2H_4-H_1$, $H_{10} = H_2+H_4$, $H_{11} = m_{11}H_5$, $H_{12} = -m_{11}H_6-H_5$, $H_{13} = H_6+m_{11}$, $H_{14} = -H_{13}-H_{10}$, $H_{15} = -H_{12}+H_{13}H_{10}-H_9$, $H_{16} = -H_{11}+H_{10}H_{12}+H_9H_{13}-H_8$, $H_{17} = H_{10}H_{11}+H_9H_{12}+H_8H_{13}-H_7$, $H_{18} = H_{11}+H_8H_{12}+H_7H_{13}$, $H_{19} = H_{11}H_8+H_7H_{12}$, $H_{20} = H_7H_{11}$, $m_{11} = -\alpha-\mu_1$, $m_{13} = -\beta_1\bar{U}$, $m_{14} = \kappa_1$, $m_{16} = -\beta_2\bar{U}$, $m_{17} = \kappa_2$, $m_{21} = \alpha$, $m_{22} = -(1-\theta)a\bar{C} - \mu_2$, $m_{32} = \phi$, $m_{33} = -\mu_3$, $m_{41} = \beta_1$, $m_{43} = \beta_1\bar{U}$, $m_{44} = -\kappa_1-\delta-\mu_4$, $m_{52} = b\bar{C}$, $m_{55} = -2\gamma_2\bar{C}-\mu_5$, $m_{65} = \psi$, $m_{66} = -\mu_6$, $m_{74} = \delta$, $m_{76} = \beta_2\bar{U}$, $m_{77} = -\kappa_2-\eta$.

Following Routh Hurwitz criteria $\bar{P}$ will be locally asymptotically stable if and only if

$$\begin{cases} Q_1 > 0,\ Q_3 > 0,\ Q_5 > 0,\ Q_7 > 0, \\ b_1 = \frac{Q_1Q_2-Q_3}{Q_1} > 0,\ b_2 = \frac{Q_1Q_4-Q_5}{Q_1} > 0,\ b_3 = \frac{Q_1Q_6-Q_7}{Q_1} > 0,\ c_1 = \frac{Q_3b_1-Q_1b_2}{b_1} > 0, \\ c_2 = \frac{b_1Q_5-Q_1b_3}{b_1} > 0,\ c_3 = Q_7 > 0,\ d_1 = \frac{b_2c_1-b_1c_2}{c_1} > 0,\ d_2 = \frac{c_1b_3-b_1c_3}{c_1} > 0, \\ e_1 = \frac{d_1c_2-c_1d_2}{d_1} > 0,\ e_2 = Q_7 > 0,\ f_1 = \frac{d_2e_1-d_1e_2}{e_1} > 0. \end{cases} \tag{3.5}$$

Thus the proposition is proved. □

Proposition 3.4. *Assume that the MSME equilibrium P^* exists satisfying the conditions stated in* (3.3)*. Then P^* will be locally asymptotically stable if and only if the conditions determined in* (3.7) *are satisfied.*

Proof. The characteristic equation of the jacobian matrix associated with model system (2.1) evaluated at P^* can be written as

$$\xi^7 + P_1\xi^6 + P_2\xi^5 + P_3\xi^4 + P_4\xi^3 + P_5\xi^2 + P_6\xi + P_7 = 0, \tag{3.6}$$

where $P_1 = -D_1$, $P_2 = -D_2$, $P_3 = -E_1$, $P_4 = -E_2$, $P_5 = -E_3$, $P_6 = -E_4$, $P_7 = -E_5$, $D_1 = C_{15} - C_{11}$, $D_2 = C_{16} + C_{11}C_{15} - C_{12}$, $D_3 = C_{17}C_{16}C_{11} + C_{12}C_{15} - C_{13}$, $D_4 = C_{17}C_{11} + C_{12}C_{16} + C_{13}C_{15} - C_{14}$, $D_5 = C_{12}C_{17} + C_{13}C_{16} + C_{14}C_{15}$, $D_6 = C_{13}C_{17} + C_{14}C_{17}$, $D_7 = C_{14}C_{17}$, $E_1 = D_3 - I_{29}$, $E_2 = D_4 + A_{29} - C_6$, $E_3 = D_5 + I_{30} + C_7$, $E_4 = D_6 + I_{31} + C_8$, $D_7 + I_{32} + C_9$, $I_1 = n_{33}n_{21}n_{13}$, $I_2 = n_{55}n_{66}$, $I_3 = n_{55} + n_{66}$, $I_4 = n_{47}n_{74}$, $I_5 = -n_{33}n_{21}n_{43}$, $I_6 = n_{44}n_{77} - I_4$, $I_7 = n_{44}n_{77}$, $I_8 = n_{14}n_{77}$, $I_9 = n_{14}$, $I_{10} = n_{17}n_{74}$, $I_{11} = I_1I_2I_6$, $I_{12} = I_1I_2I_7$, $I_{13} = I_1I_2$, $I_{14} = I_1I_3I_6$, $I_{15} = I_1I_3I_7$, $I_{16} = I_1I_3$, $I_{17} = I_1I_6$, $I_{18} = I_1I_7$, $I_{19} = I_2I_5I_8$, $I_{20} = I_2I_5I_9$, $I_{21} = I_2I_5I_{10}$, $I_{22} = I_3I_5I_8$, $I_{23} = I_3I_5I_9$, $I_{24} = I_3I_5I_{10}$, $I_{25} = I_5I_8$, $I_{26} = I_5I_9$, $I_{27} = I_5I_{10}$, $I_{28} = I_1$, $I_{29} = -I_{16}-I_{18}-I_{26}$, $I_{30} = I_{13}+I_{15}+I_{17}+I_{23}+I_{25}-I_{27}$, $I_{31} = I_{12}-I_{14}-I_{20}-I_{22}+I_{24}$, $I_{32} = I_{11}I_{19}-I_{21}$, $C_1 = n_{33}n_{66}$, $C_2 = n_{33} + n_{66}$, $C_3 = n_{55}n_{22}$, $C_4 = n_{55} + n_{22}$, $C_5 = n_{47}n_{74}$, $C_6 = n_{44}n_{77}$, $C_7 = n_{44} + n_{77}$, $C_8 = C_6 - C_7$, $C_9 = -n_{41}n_{14}n_{77} + n_{41}n_{17}n_{74} + n_{71}n_{14}n_{47} - n_{71}n_{17}n_{44}$, $C_{10} = n_{14}n_{41} - n_{17}n_{71}$, $C_{11} = -C_4 - C_2$, $C_{12} = C_3 - C_2C_4 + C_1$, $C_{13} = -C_2C_3 - C_1C_4$, $C_{15} = C_7 + n_{11}$, $C_{16} = C_{10} - C_8 - C_7n_{11}$, $C_{17} = C_9 + C_8n_{11}$, $n_{11} = -\alpha - \beta_1J_E^* - \beta_2J_C^* - \mu_1$, $n_{13} = \beta_1U^*$, $n_{14} = \kappa_1$, $n_{16} = -\beta_2U^*$, $n_{17} = \kappa_2$, $n_{21} = \alpha$, $n_{22} = -2\gamma_1E^* - (1-\theta)aC^* - \mu_2$, $n_{25} = -(1-\theta)aE^*$, $n_{32} = \phi$, $n_{33} = -\mu_3$, $n_41 = \beta_1$, $n_{43} = \beta_1U^*$, $n_{44} = -\kappa_1 - \delta = \mu_4$, $n_{47} = \eta$, $n_{52} = bC^*$, $n_{55} = -2\gamma_2C^* - bE^* - \mu_5$, $n_{65} = \psi$, $n_{66} = -\mu_6$, $n_{71} = \beta_2J_c^*$, $n_{76} = \beta_2U^*$, $n_{77} = -\kappa_2 - \eta$.

According to the Routh Hurwitz criteria P^* will be locally asymptotically stable if and only if

$$\begin{cases} P_1 > 0,\ P_3 > 0,\ P_5 > 0,\ P_7 > 0, \\ b_1^* = \frac{P_1P_2-P_3}{P_1} > 0,\ b_2^* = \frac{P_1P_4-P_5}{P_1} > 0,\ b_3^* = \frac{P_1P_6-P_7}{P_1} > 0,\ c_1^* = \frac{P_3b_1^*-P_1b_2^*}{b_1^*} > 0, \\ c_2^* = \frac{b_1^*P_5-P_1b_3^*}{b_1^*} > 0,\ c_3^* = P_7 > 0,\ d_1^* = \frac{b_2^*c_1^*-b_1^*c_2^*}{c_1^*} > 0,\ d_2^* = \frac{c_1^*b_3^*-b_1^*c_3^*}{c_1^*} > 0, \\ e_1^* = \frac{d_1^*c_2^*-c_1^*d_2^*}{d_1^*} > 0,\ e_2^* = P_7 > 0,\ f_1^* = \frac{d_2^*e_1^*-d_1^*e_2^*}{e_1^*} > 0. \end{cases} \tag{3.7}$$

This proves the proposition. □

4. Numerical simulations

In this section we perform parameter estimation and time series analysis of the model system (2.1) with respect to important and sensitive system parameters using MATLAB R2020a software.

4.1. Parameter estimation

First, we estimate the parameters of the model system (2.1) with respect to the secondary data of the employments generated by the MSMEs in India as detailed in Table 1.

End Year	Total Active MSMEs (Lakh No)	Employment Generated (in Lakh person)
1991	67.85	158.36
1992	70.65	166.99
1993	73.57	174.86
1994	76.51	182.65
1995	79.62	191.5
1996	82.85	197.34
1997	86.22	205.89
1998	89.75	213.18
1999	93.98	220.56
2000	97.16	229.15
2001	101.11	238.75
2002	105.25	249.36
2003	109.51	260.25
2004	114.95	271.44
2005	118.69	282.57
2006	123.45	299.87
2007	361.76	805.24
2008	377.36	842
2009	393.7	880.86
2010	410.8	921.82
2011	428.73	966.18
2012	447.64	1011.71
2013	467.54	1061.44
2014	488.46	1114.3
2015	510.57	1171.33
2016	633.88	1109.89
2017	Data Not Available	Data Not Available
2018	Date Not Available	Data Not Available
2019	Date Not Available	Data Not Available
2020	Data Not Available	65.64458
2021	Data Not Available	112.27745
2022	Data Not Available	130.19919
Up to 07.12.2022	Data Not Available	8.423452

Table 1. Year wise active MSMEs and employment generated in the MSME sector in India (*cf.* [3, 16])

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It should be noted that the amount of employment generated by the MSMEs in India has a huge jump between the year 2006 and 2007. From Table 1 one can easily see that, in 2006 a total of 299.87 lakh no jobs were created in MSME sector of India. And the same for the year 2007 was 805.24. We therefore, initially fit the employment generated data for the duration 1991 - 2006 with respect to the system variable J_E that represents the number of employment generated by the MSME entrepreneurship ventures in model system (2.1) using least mean square error method. According to World Bank Data, in 1991 the population of India was 888.9 lakh and national unemployment rate was 5.6%. Hence the initial value for the system variable $U(t)$, the variable measuring unemployed individuals, has been considered as $U(0) = 49.7784$ lakh. The estimated parameter values are found to be:

$$\begin{aligned}
&\Delta = 53.22562,\ \alpha = 0.013421,\ \beta_1 = 0.00015445,\ \beta_2 = 0.000166234,\\
&\kappa_1 = 0.012361,\ \kappa_2 = 0.00188723,\ \gamma_1 = 0.00266342,\ \gamma_2 = 0.00213346,\\
&\theta = 0.28823,\ a = 0.055342,\ b = 0.00588376,\ \phi = 0.1231,\\
&\delta = 0.06673,\ \eta = 0.003863,\ \psi = 0.222611,\ \Gamma = 0.037742,\\
&\mu_1 = 0.0011121,\ \mu_2 = 0.012771,\ \mu_3 = 0.013251,\ \mu_4 = 0.0511234,\\
&\mu_5 = 0.001223,\ \mu_6 = 0.0011156,\ \mu_7 = 0.00121.
\end{aligned} \tag{4.1}$$

The data fit of model system (2.1) for the estimated parameter values given in (4.1) is depicted in Figure 2.

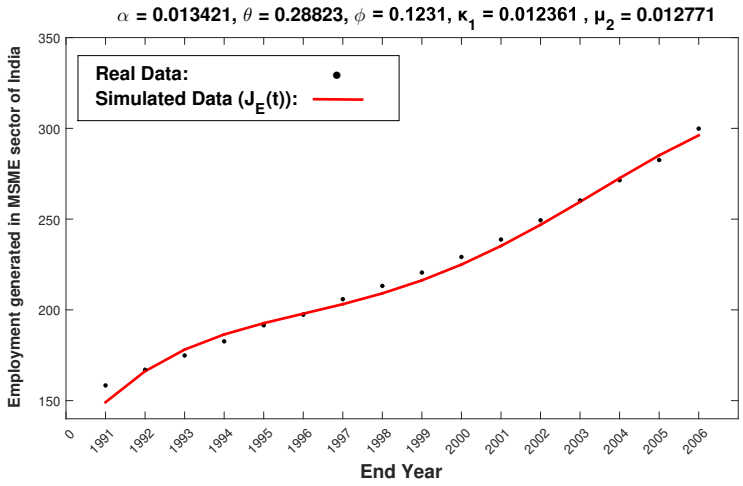

Figure 2. Fitting of the "Employment Generated" data for the period 1991 - 2006 as listed in Table 1 with respect to the system variable $J_E(t)$ of the model system (2.1). Parameters are as in (4.1)

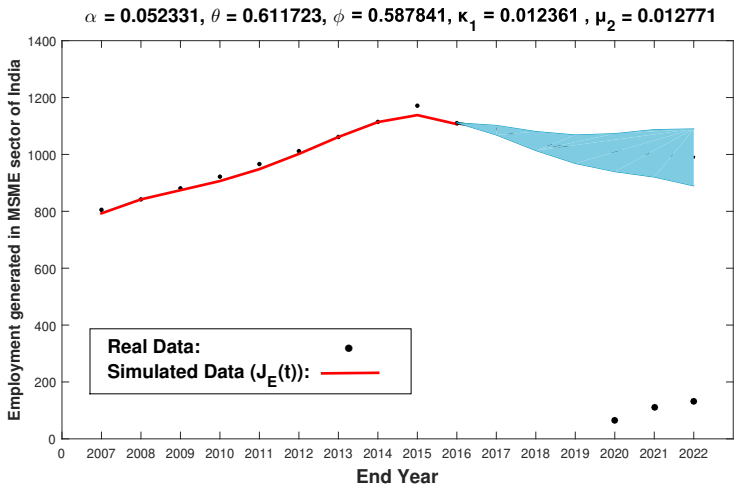

Figure 3. Fitting of the "Employment Generated" data for the period 2007 - 2022 as mentioned Table 1 with respect to system variable $J_E(t)$ of model system (2.1). Here $\alpha = 0.052331$, $\theta = 0.611723$, $\phi = 0.587841$ and other parameters are as in Figure 2

From Figure 2 it can be easily seen that our model shows a good fit with real "Employment Generated" data for the period 1991 - 2006. However, for the estimated parameter values as mentioned in (4.1), the model did not yield a good fit with the real data for the period 2007 - 2016. In fact, it could not account for the jump between the real data in 2006 and 2007 as mentioned earlier. But when we increased α, θ and ϕ from their previously estimated values to $\alpha = 0.052331$, $\theta = 0.611723$ and $\phi = 0.587841$, keeping all other parameters fixed as in (4.1), our model provided a good fit with the real "Employment Generated" data and simulated data with respect to the system variable $J_E(t)$ for the period 2007 - 2016. This case is shown in Figure 3. Moreover, we have drawn the predicted data, for the period 2017 - 2022 in this diagram with 95% confidence interval (shaded area). This shows that the jump in MSME sector was due to increased governmental support (represented by the parameter θ), more skill development programs to the youth (represented by the parameter α) and rate of creation of new jobs in MSME sector at a higher rate (represented by the parameter ϕ). The real data of the employment generated in MSME sector of India in the period 2017 - 2018 is unavailable (see Table 1). However, our model did not fit with the available data for the period 2020 - 2022. There is a huge deviation from the real data and simulated data that can clearly be seen in Figure 3. We assert that, this is due to the sudden outbreak of COVID - 19 pandemic. This global pandemic has adversely affected the MSME sector of India as discussed in Section 1.4. Now to fit our model with real data for the period 2020 - 2022 we again adjust the parameter values to understand the impacts of COVID-19 on the employment generation in MSME sector of India. It is evident that in time of COVID-19 pandemic, due to consecutive lock downs a lot of MSMEs lost their business and were shut down. Eventually, the associated employment opportunities were lost. A lot of people, working in the MSME sector lost their jobs. Therefore, increasing κ_1, the parameter that represents the rate at which individuals working in MSME companies lose their jobs and μ_2, the rate at which MSMEs are shut down, would be a good option to capture the impact of COVID-19 on the MSME sector. We increased the previously estimated value of κ_1 as listed in (4.1) to $\kappa_1 = 0.462217$ to incorporate the loss of jobs in MSME sector during COVID-19 pandemic. We further increased μ_2 as listed in (4.1) to $\mu_2 = 0.0611755$ to factor in the closure of MSME ventures during COVID-19 pandemic. With these modifications our model simulated data provides a good fit with real data when all other parameters are kept same as in Figure 3. The data fit diagram for the duration 2020 - 2022 with respect to the system variable J_E is given in Figure 4.

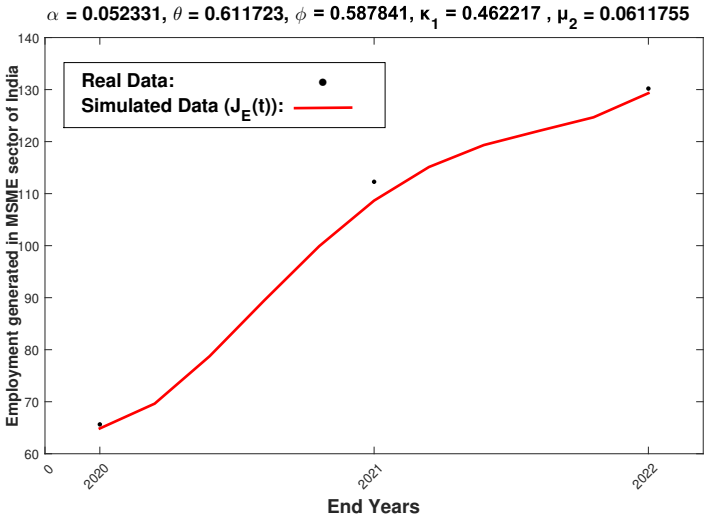

Figure 4. Fitting of the "Employment Generated" data for the period 2020 - 2022 as mentioned Table 1 with respect to system variable $J_E(t)$ of model system (2.1). Here $\kappa_1 = 0.462217$, $\mu_2 = 0.0611755$ and other parameters are as in Figure 3

4.2. Time series analysis

We have performed the Partial Rank Correlation Coefficient (PRCC) analysis of the system parameters to determine their sensitivity. The corresponding PRCC diagram is shown in Figure 5. From Figure 5 we can easily see that the parameters α, θ, ϕ, κ_1 and μ_2 are the most sensitive parameters that can alter the system dynamics significantly.

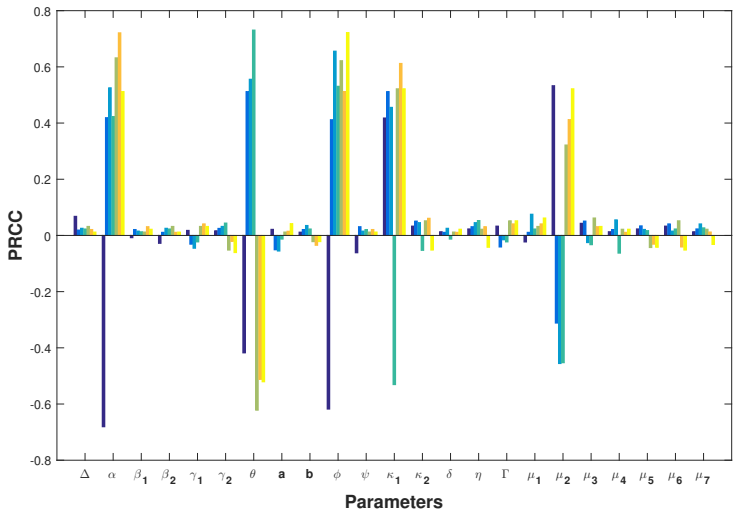

Figure 5. PRCC sensitivity analysis of the parameters of model system (2.1) with $p - value < 0.0001$

4.2.1. Effect of the parameter α

The parameter α measures the rate of training and skills acquired by an unemployed individual to start an entrepreneurship in MSME sector. Now we consider the parameter values (4.1) to study the effect of the parameter α. For those parameter values the time evolution diagram of $U(t)$ (the unemployed individuals) and $S_E(t)$ (individuals employed in MSME sector) have been drawn in Figure 6(a). In this case system converges to stable E^* and it shows the number of unemployed individuals is higher than the MSME employed individuals. However, if we increase α to 0.05 (Figure 6(b)) then the number of unemployed individuals decrease and the same for MSME employed individuals increase above the unemployed individuals. This signifies that if the unemployed youth is given proper skill development training in various business opportunities in MSME sector then they can successfully run MSME entrepreneurships with governmental financing schemes and facilities. Thus, they can generate more jobs through their MSMEs which on the other hand will reduce the number of unemployed people.

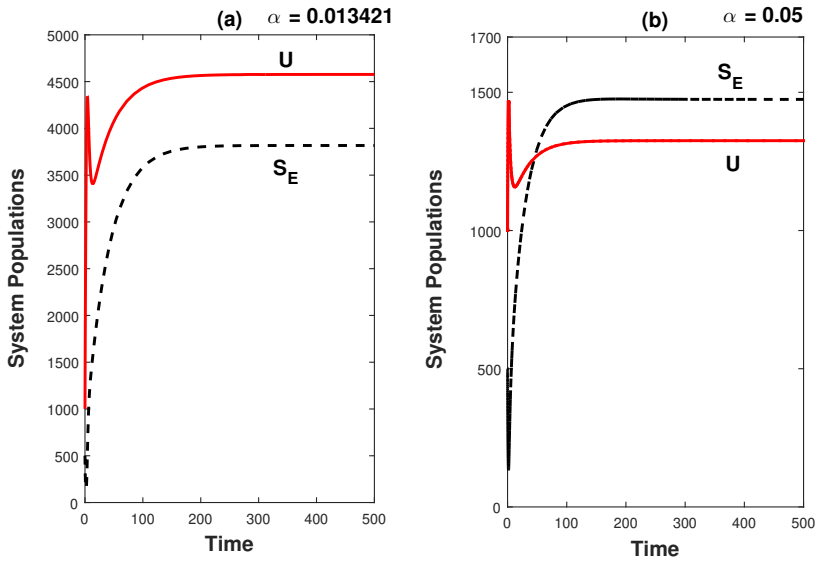

Figure 6. Time evolution of population $U(t)$ and $S_E(t)$ of model system (2.1) for (a) $\alpha = 0.013421$ and (b) $\alpha = 0.05$. Other parameters are as in (4.1)

4.2.2. *Effect of the parameter θ*

The effect of the available governmental support for the MSMEs is represented by the parameter θ. This support may be, financial or other necessary facilities provided to the MSMEs to ease their business ventures. The effect of θ on system dynamics is depicted in Figure 7. First, we have drawn the time evolution diagram of $U(t)$ and $S_E(t)$ in Figure 7(a) for $\theta = 0.28823$ as estimated in (4.1). It shows that, for almost 29% governmental support, the number of unemployed is more then the number of individuals employed in MSME sector. However, if we increase θ to 0.67, then we can see that with 67% governmental support, the number of MSME employed individuals surpasses the unemployed (Figure 7(b)). Therefore, the governmental support plays an important role in development of MSMEs and thereby to control the unemployment. In both the cases, the system trajectories converge to stable E^*.

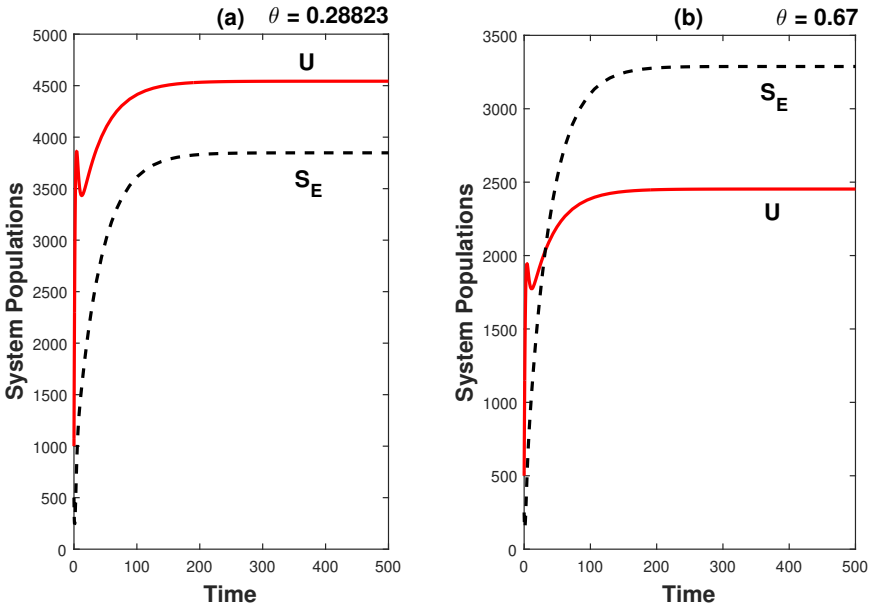

Figure 7. Time evolution of population $U(t)$ and $S_E(t)$ of model system (2.1) for (a) $\theta = 0.28823$ and (b) $\theta = 0.67$. Other parameters are as in (4.1)

4.2.3. *Effect of the parameter ϕ*

The parameter ϕ measures the rate at which new jobs are being created in MSME sector. This parameter is highly significant as the control mechanism of unemployment problem, is directly correlated with the generation of new jobs. First to study the effect of ϕ we consider the estimated value $\phi = 0.1231$ from (4.1). We have drawn the time evolution diagram of $U(t)$ and $S_E(t)$ population in Figure 8(a). It shows for the parameter values as in (4.1), system converges to stable E^* but unemployment level is more than the employed level. But if we increase ϕ to 0.35, then unemployment reduces and the density of the individuals employed in MSME sector increases. This case is shown in Figure 8(b). Hence, we assert that increasing employment opportunities in MSME sector plays an important role to resolve the issue of unemployment.

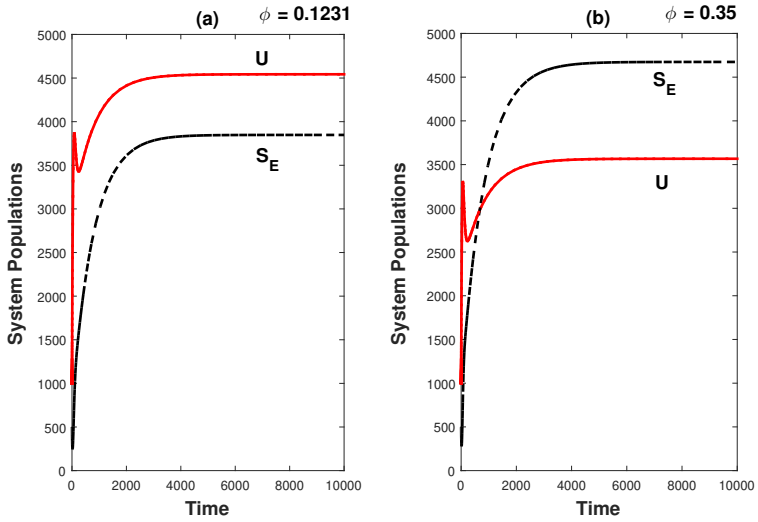

Figure 8. Time evolution of population $U(t)$ and $S_E(t)$ of model system (2.1) for (a) $\phi = 0.1231$ and (b) $\phi = 0.35$. Other parameters are as in (4.1)

4.2.4. *Effect of the parameters κ_1 and μ_2*

As discussed earlier, the effect of these two parameters, namely κ_1 that measures the rate at which the MSME employees lose their jobs and μ_2 which represents the rate at which MSMEs are shut down due to financial or other sustainability issues, play significant roles to analyze the impacts of COVID - 19 pandemic on MSME sector in India. Both of them are negatively correlated with the employment generation in MSME sector. If κ_1 is taken at its base value 0.012361 as mentioned (4.1), then system trajectories converge to stable E^* and level of unemployment remains greater than the level of employment generated in MSME sector (Figure 9(a)). However, if κ_1 is increased further to $\kappa_1 = 0.1$, then we can understand the scenario which appeared during the lockdowns of COVID - 19 pandemic. During this time lots and lots of people lost their jobs in MSME sector, unemployment increased and employment level decreased considerably. This situation is depicted in Figure 9(b).

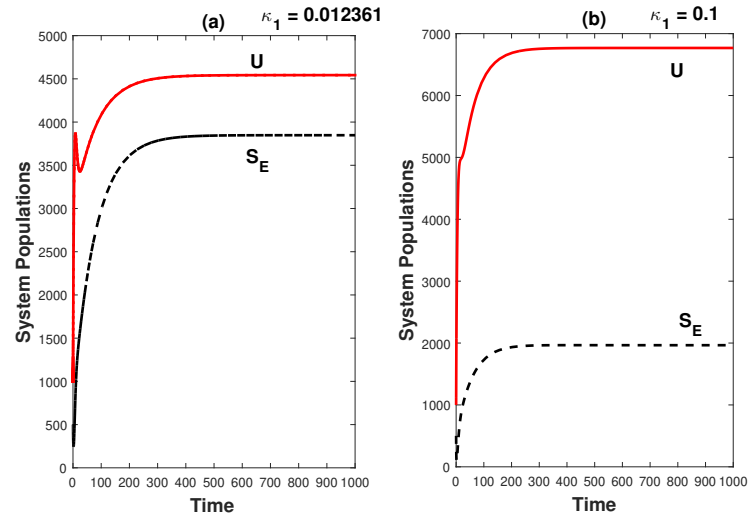

Figure 9. Time evolution of population $U(t)$ and $S_E(t)$ of model system (2.1) for (a) $\kappa_1 = 0.012361$ and (b) $\kappa_1 = 0.1$. Other parameters are as in (4.1)

Similar analysis can be given for μ_2. When μ_2 is increased from its base level (Figure 10(a)) to a higher level, signifying the shut down of a significant number of MSMEs during COVID - 19 lockdown, unemployment increased many folds (Figure 10(b)).

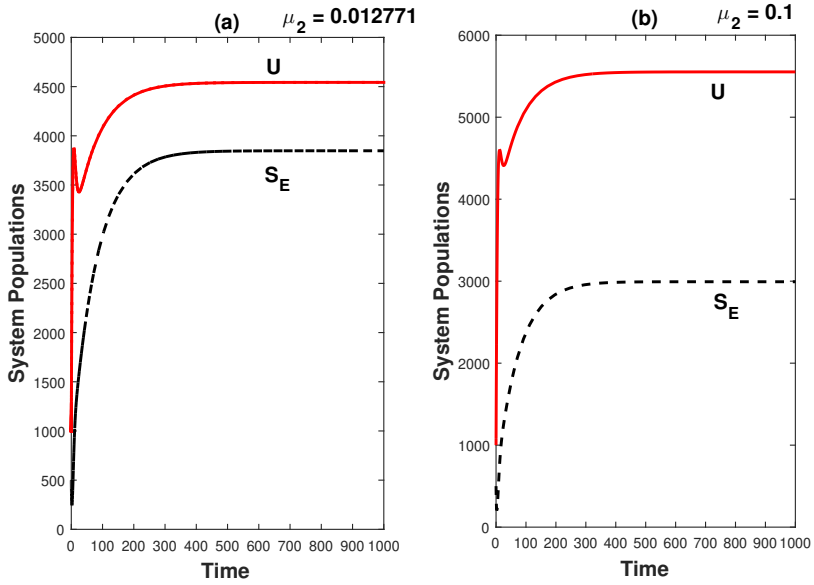

Figure 10. Time evolution of population $U(t)$ and $S_E(t)$ of model system (2.1) for (a) $\mu_2 = 0.012771$ and (b) $\mu_2 = 0.1$. Other parameters are as in (4.1)

5. Conclusion

In our study we have proposed a deterministic mathematical model with the help of ordinary differential equations to analyze the impact of MSMEs to resolve the issue of unemployment. We have determined the positivity & boundedness of the system solutions, conditions for analytical existence of equilibrium points and their local asymptotic stability. To validate our model, we have performed estimation of system parameters using existing secondary data. It has been found that our model shows a good fit with the real data for the estimated parameter values during the period 1991 to 2006. The fluctuations of the real data after 2006 has been explained by adjusting the suitable parameters. It has been found that this fluctuation is the joint effect of increasing governmental support towards MSMEs, initiation of new youth skill development programs and generation of more jobs in MSME sector. We have also performed prediction analysis with 95% confidence interval for the years 2017 to 2019 as data for this time period is not available. But our prediction analysis did not match the data for the years 2020 to 2022. However, by adjusting the estimated parameter values and fitting our model with real available data we have shown that, this failure of prediction was due to sudden and huge job loss in the MSMEs sector during COVID-19 pandemic. During COVID-19 pandemic period lots of MSMEs were shut down abruptly due to consecutive lockdowns and that caused the nose dive in the number of jobs in MSME sector. We have also performed the sensitivity analysis of the system parameters using PRCC analysis. It has been found that the parameters representing the effect of skill development and training programs to provide necessary skills to unemployed individuals to start MSME entrepreneurship ventures, governmental support to the MSMEs and higher rate of job creation in MSME sector can efficiently regulate the system dynamics. In fact, they are positively correlated to resolve the issue of unemployment. Whereas, the parameters representing the loss of jobs in MSME sector and closing of MSMEs due to financial crisis or other sustainability issues negatively impact any feasible solution to the unemployment problem. Moreover, through time series analysis, we have shown that the fluctuations in the number of employment generated in MSME sector over the years can be explained by variation of the parameters measuring these above mentioned aspects. Therefore, the results of our analysis can be used while

forming various policy decisions, short term or long term for development of entrepreneurship start ups in MSME sector in the perspective of resolving the issue of unemployment in India.

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