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Topological indices on splitting, shadow and total graphs

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Abstract

In mathematical chemistry, topological indices play a vital role. Most of the topological indices are defined on well known graph concepts such as degree of a vertex, distance, eccentricity of a vertex etc. Here we discuss degree based topological indices. In this paper, we compute the forgotten index and the Zagreb index of degree based topological indices for splitting, shadow and total graphs.

Keywords: Zagreb indices, forgotten index, shadow graph

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1. Introduction

Numerous varieties exist for topological indices, including degree based and distance based indices. The earliest known distance-based topological index is the Wiener index [22], which is calculated as the half-sum of the distances between each pair of vertices in a molecular network (*cf.* [21]). Numerous topological indices have been studied in QSPR and QSAR studies.

The degree based first and second Zagreb indices were initially presented in [7], and further information about them is available in [9]. These Zagreb indices, which are represented by M_1 and M_2, are as follows:

$$M_1 = M_1(G) = \sum_{\alpha \in V(G)} d_\alpha^2$$

and

$$M_2 = M_2(G) = \sum_{\alpha\beta \in E(G)} d_\alpha d_\beta$$

respectively. Several authors have examined some recent findings about these indices (*cf.* [2, 8, 10, 23]).

Following the first and second Zagreb indices, Furtula and Gutman [3] proposed the forgotten topological index (also known as the F-index), that was defined as,

$$FT(G) = \sum_{\alpha \in V(G)} d_\alpha^3 = \sum_{\alpha\beta \in E(G)} \left[d_\alpha^2 + d_\beta^2 \right].$$

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where d_α is degree of vertices α. Furtula and Gutman found that the forgotten topological index had virtually the same prediction ability as the first Zagreb index, as well as the eccentric factor and entropy, and that both have correlation values more than 0.95. This point explains why the forgotten topological index is beneficial for assessing the chemical and physiological aspects of drug molecule structures. Gao et al. recently published the neglected topological index of certain major pharmacological molecular structures (*cf.* [4]). Furthermore, many researchers are continuing to investigate the forgotten topological index through various studies (*cf.* [1, 5, 6], [10]-[13], [16, 17, 19]).

For every vertex $\alpha \in V(G)$, the open neighbourhood set $N(\alpha)$ is the set of all vertices adjacent to α in G. For a graph G the *splitting graph* [14, 18] $S^{'}(G)$ of a graph G is obtained by adding a new vertex $\alpha^{'}$ corresponding to each vertex α of G such that $N(\alpha) = N(\alpha^{'})$. The *shadow graph* [14, 18] $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say $G^{'}$ and $G^{''}$. Join each vertex $\alpha^{'}$ in $G^{'}$ to the neighbours of corresponding vertex $\alpha^{''}$ in $G^{''}$. The *total graph* [15] $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G. In this paper, we compute the forgotten index and Zagreb index of splitting, shadow and total graphs.

2. Splitting graphs

In this section we find the F-index and Zagreb index of the splitting graphs of star, bistar, path, and cycle.

Theorem 2.1. *The F-index and Zagreb of splitting graph of star is*

$$FT(S^{'}(K_{1,t})) = 9t(t^2+1),$$

$$M_1(S^{'}(K_{1,t})) = 5t(t+1),$$

$$M_2(S^{'}(K_{1,t})) = 5t^2.$$

Proof. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the pendant vertices and α be the apex vertex of $K_{1,t}$ and $\beta, \beta_1, \beta_2, \ldots, \beta_t$ are added vertices corresponding to $\alpha, \alpha_1, \alpha_2, \ldots, \alpha_t$ to obtain $S^{'}(K_{1,t})$.

Firstly, we select a vertex α on $S^{'}(K_{1,t})$ of degree t. There are t vertices $\alpha_1, \alpha_2, \ldots, \alpha_t$ of degree 1 which are adjacent to α. Since the degree of α is $2t$, the other vertices which are adjacent to α is $\beta_1, \beta_2, \ldots, \beta_t$ of degree $2t$. Now, we select the t vertices $\beta_1, \beta_2, \ldots, \beta_t$ of degree 2 which are adjacent to β. Therefore degree of a vertex β is t. Hence we can compute the F-index and Zagreb index of $S^{'}(K_{1,t})$ as

$$\begin{aligned} FT(S^{'}(K_{1,t})) &= d_\alpha^3 + d_\beta^3 + \sum_{i=1}^{t} d_{\alpha_i}^3 + \sum_{i=1}^{t} d_{\beta_i}^3 \\ &= 9t(t^2+1), \\ M_1(S^{'}(K_{1,t})) &= d_\alpha^2 + d_\beta^2 + \sum_{i=1}^{t} d_{\alpha_i}^2 + \sum_{i=1}^{t} d_{\beta_i}^2 \\ &= 5t(t+1), \\ M_2(S^{'}(K_{1,t})) &= d_\alpha \sum_{i=1}^{t} d_{\alpha_i} + d(\alpha) \sum_{i=1}^{t} d_{\beta_i} + d_\beta \sum_{i=1}^{t} d_{\beta_i} \\ &= 5t^2. \end{aligned}$$

□

Theorem 2.2. *The F-index and Zagreb of splitting graph of bistar is*

$$FT(S^{'}(B_{t,t})) = 2(2t+2)^3 + 2(t+1)^3 + 18t,$$

$$M_1(S^{'}(B_{t,t})) = 2(2t+2)^2 + 2(t+1)^2 + 10t,$$

$$M_2(S^{'}(B_{t,t})) = (2t+2)(10t+4) + 4t(t+1).$$

Proof. Let $\beta, \alpha, \beta_1, \beta_2, \ldots, \beta_t, \alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of $B_{t,t}$. Let $\beta^{'}, \alpha^{'}, \beta^{'}_i, \alpha^{'}_i$ are added vertices corresponding to $\beta, \alpha, \beta_i, \alpha_i$ to obtain $S^{'}(B_{t,t})$. Then the degree of each vertex of $S^{'}(B_{t,t})$ can be defined as

$$\begin{aligned} d_\beta &= d_\alpha = 2t + 2, \\ d_{\beta'} &= d_{\alpha'} = t + 1, \\ d_{\beta_i} &= d_{\alpha_i} = \sum_{i=1}^{t} 2, \\ d_{\beta'_i} &= d_{\alpha'_i} = \sum_{i=1}^{t} 1. \end{aligned}$$

Hence we can obtain the F-index and Zagreb index of $S^{'}(B_{t,t})$ as

$$\begin{aligned} FT(S^{'}(B_{t,t})) &= d^3_\beta + d^3_\alpha + d^3_{\beta'} + d^3_{\alpha'} + \sum_{i=1}^{t} d^3_{\alpha_i} + \sum_{i=1}^{t} d^3_{\beta_i} + \sum_{i=1}^{t} d^3_{\alpha'_i} + \sum_{i=1}^{t} d^3_{\beta'_i} \\ &= 2(2t + 2)^3 + 2(t + 1)^3 + 18t, \\ M_1(S^{'}(B_{t,t})) &= d^2_\beta + d^2_\alpha + d^2_{\beta'} + d^2_{\alpha'} + \sum_{i=1}^{t} d^2_{\alpha_i} + \sum_{i=1}^{t} d^2_{\beta_i} + \sum_{i=1}^{t} d^2_{\alpha'_i} + \sum_{i=1}^{t} d^2_{\beta'_i} \\ &= 2(2t + 2)^2 + 2(t + 1)^2 + 10t, \\ M_2(S^{'}(B_{t,t})) &= d_\beta d_\alpha + d_{\beta'} d_\alpha + d_\beta d_{\alpha'} + d_{\beta'} \sum_{i=1}^{t} d_{\beta_i} + d_\beta \sum_{i=1}^{t} d_{\beta_i} + d_\beta \sum_{i=1}^{t} d_{\beta'_i} + d_{\alpha'} \sum_{i=1}^{t} d_{\alpha_i} + d_\alpha \sum_{i=1}^{t} d_{\alpha_i} + d_\alpha \sum_{i=1}^{t} d_{\alpha'_i} \\ &= (2t + 2)(10t + 4) + 4t(t + 1). \end{aligned}$$

□

Theorem 2.3. *The F-index and Zagreb index of splitting graph of path is*

$$FT(S^{'}(P_t)) = 72(t - 2) + 18,$$

$$M_1(S^{'}(P_t)) = 20t - 30,$$

$$M_2(S^{'}(P_t)) = 24(t - 2)^2 + 8(t - 3)^2 + 32.$$

Proof. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of the path P_t. Let $E(S^{'}(P_t)) = \{\alpha_i\alpha^{'}_{i+1}, \alpha^{'}_i\alpha_{i+1}, \alpha_i\alpha_{i+1} : 1 \leq i \leq t-1\}$. Besides, the degree of each vertex of $S^{'}(P_t)$ can be determined as

$$\begin{aligned} d_{\alpha_1} &= d_{\alpha_t} = 2, \\ d_{\alpha'_1} &= d_{\alpha'_t} = 1, \\ d_{\alpha_i} &= \sum_{i=2}^{t-1} 4, \\ d_{\alpha'_i} &= \sum_{i=2}^{t-1} 2. \end{aligned}$$

Hence we can obtain the forgotten and Zagreb index of $S^{'}(P_t)$ as

$$\begin{aligned} FT(S^{'}(P_t)) &= d^3_{\alpha_1} + d^3_{\alpha_t} + d^3_{\alpha^{'}_1} + d^3_{\alpha^{'}_t} + \sum_{i=2}^{t-1} d^3_{\alpha_i} + \sum_{i=2}^{t-1} d^3_{\alpha^{'}_i} \\ &= 72(t-2) + 18, \\ M_1(S^{'}(P_t)) &= d^2_{\alpha_1} + d^2_{\alpha_t} + d^2_{\alpha^{'}_1} + d^2_{\alpha^{'}_t} + \sum_{i=2}^{t-1} d^2_{\alpha_i} + \sum_{i=2}^{t-1} d^2_{\alpha^{'}_i} \\ &= 20t - 30, \\ M_2(S^{'}(P_t)) &= \sum_{i=1}^{t-1} d_{\alpha_i} d_{\alpha_{i+1}} + \sum_{i=1}^{t-1} d_{\alpha^{'}_i} d_{\alpha_{i+1}} + \sum_{i=1}^{t-1} d_{\alpha_i} d_{\alpha^{'}_{i+1}} \\ &= 24(t-2)^2 + 8(t-3)^2 + 32. \end{aligned}$$

□

Theorem 2.4. *The F-index and Zagreb of splitting graph of cycle is*

$$FT(S^{'}(C_t)) = 72t,$$

$$M_1(S^{'}(C_t)) = 20t,$$

$$M_2(S^{'}(C_t)) = 32(t^2 + 1).$$

Proof. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of the cycle C_t. Let

$$E(S^{'}(C_t)) = \{\alpha_i\alpha^{'}_{i+1}, \alpha^{'}_i\alpha_{i+1}, \alpha_i\alpha_{i+1} : 1 \le i \le t-1\} \{\alpha_t\alpha_1, \alpha^{'}_1\alpha_t, \alpha^{'}_t\alpha_1\}.$$

Furthermore, the degree of each vertex of $S^{'}(C_t)$ can be determined as

$$\begin{aligned} d_{\alpha_i} &= \sum_{i=1}^{t} 4, \\ d_{\alpha^{'}_i} &= \sum_{i=1}^{t} 2. \end{aligned}$$

Hence we can obtain the forgotten and Zagreb index of $S^{'}(C_t)$ as

$$\begin{aligned} FT(S^{'}(C_t)) &= \sum_{i=1}^{t} d^3_{\alpha_i} + \sum_{i=1}^{t} d^3_{\alpha^{'}_i} \\ &= 72t, \\ M_1(S^{'}(C_t)) &= \sum_{i=1}^{t} d^2_{\alpha_i} + \sum_{i=1}^{t} d^2_{\alpha^{'}_i} \\ &= 20t, \\ M_2(S^{'}(C_t)) &= d_{\alpha_t} d_{\alpha_1} + \sum_{i=1}^{t-1} d_{\alpha_i} d_{\alpha_{i+1}} + \sum_{i=1}^{t-1} d_{\alpha^{'}_i} d_{\alpha_{i+1}} + d_{\alpha^{'}_1} d_{\alpha_t} + \sum_{i=1}^{t} d_{\alpha_i} d_{\alpha^{'}_{i+1}} + d_{\alpha^{'}_t} d_{\alpha_1} \\ &= 32(t^2 + 1). \end{aligned}$$

□

3. Shadow graphs

We find the F-index and Zagreb index of the shadow graphs of star, bistar, path, and cycle in this section.

Theorem 3.1. *The F-index and Zagreb index of shadow graph of star is*

$$FT(D_2(K_{1,t})) = 16t(t^2 + 1),$$

$$M_1(D_2(K_{1,t})) = 8t(t + 1),$$
$$M_2(D_2(K_{1,t})) = 16t.$$

Proof. Let $\beta, \beta_1, \beta_2, \ldots, \beta_t$ and $\alpha, \alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of two copies of $K_{1,t}$ and

$$E(D_2(K_{1,t})) = \{\beta\beta_i, \alpha\alpha_i, \beta\alpha_i, \alpha\beta_i : 1 \le i \le t\}.$$

Then
$$|V(D_2(K_{1,t}))| = 2t + 2 \quad \text{and} \quad |E(D_2(K_{1,t}))| = 4t.$$

In addition, the degree of each vertex of $D_2(K_{1,t})$ can be given as

$$\begin{aligned} d_\beta &= d_\alpha = 2t, \\ d_{\alpha_i} &= d_{\beta_i} = \sum_{i=1}^{t} 2. \end{aligned}$$

Hence we can determine the forgotten and Zagreb index of $D_2(K_{1,t})$ as

$$\begin{aligned} FT(D_2(K_{1,t})) &= d_\beta^3 + d_\alpha^3 + \sum_{i=1}^{t} d_{\beta_i}^3 + \sum_{i=1}^{t} d_{\alpha_i}^3 \\ &= 16t(t^2 + 1), \\ M_1(D_2(K_{1,t})) &= d_\beta^2 + d_\alpha^2 + \sum_{i=1}^{t} d_{\beta_i}^2 + \sum_{i=1}^{t} d_{\alpha_i}^2 \\ &= 8t(t + 1), \\ M_2(D_2(K_{1,t})) &= d_\beta \sum_{i=1}^{t} d_{\beta_i} + d_\alpha \sum_{i=1}^{t} d(\alpha_i) + d(\beta) \sum_{i=1}^{t} d(\alpha_i) + d(\alpha) \sum_{i=1}^{t} d(\beta_i) \\ &= 16t. \end{aligned}$$

□

Theorem 3.2. *The F-index and Zagreb index of shadow graph of bistar is*

$$FT(D_2(B_{t,t})) = 4((2t + 2)^3 + 8t),$$

$$M_1(D_2(B_{t,t})) = 4((2t + 2)^2 + 4t),$$
$$M_2(D_2(B_{t,t})) = 4(2t + 2)^2 + 8((2t + 2)(2t)).$$

Proof. Let $\beta, \beta_1, \beta_2, \ldots, \beta_t, \alpha, \alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of $B_{t,t}$ and

$$E(D_2(B_{t,t})) = \{\beta\alpha, \beta'\alpha, \beta\alpha', \beta'\alpha'\} \cup \{\beta\beta_i, \beta\beta_i', \beta'\beta_i, \beta'\beta_i', \alpha\alpha_i, \alpha\alpha_i', \alpha'\alpha_i, \alpha'\alpha_i' : 1 \le i \le t\}.$$

Then
$$|V(D_2(B_{t,t}))| = 4t + 4 \quad \text{and} \quad |E(D_2(B_{t,t}))| = 8t + 4.$$

Furthermore, the degree of each vertex of $D_2(B_{t,t})$ may be determined as

$$\begin{aligned} d_\beta &= d_\alpha = d_{\beta'} = d_{\alpha'} = 2t+2, \\ d_{\alpha_i} &= d_{\beta_i} = d_{\alpha_i'} = d_{\beta_i'} = \sum_{i=1}^{t} 2. \end{aligned}$$

Thus we can obtain the F-index and Zagreb index of $D_2(B_{t,t})$ as

$$\begin{aligned} FT(D_2(B_{t,t})) &= d_\beta^3 + d_\alpha^3 + d_{\beta'}^3 + d_{\alpha'}^3 + \sum_{i=1}^{t} d_{\beta_i}^3 + \sum_{i=1}^{t} d_{\alpha_i}^3 + \sum_{i=1}^{t} d_{\beta_i'}^3 + \sum_{i=1}^{t} d_{\alpha_i'}^3 \\ &= 4((2t+2)^3 + 8t), \\ M_1(D_2(B_{t,t})) &= d_\beta^2 + d_\alpha^2 + d_{\beta'}^2 + d_{\alpha'}^2 + \sum_{i=1}^{t} d_{\beta_i}^2 + \sum_{i=1}^{t} d_{\alpha_i}^2 + \sum_{i=1}^{t} d_{\beta_i'}^2 + \sum_{i=1}^{t} d_{\alpha_i'}^2 \\ &= 4((2t+2)^2 + 4t), \\ M_2(D_2(B_{t,t})) &= d_\beta d_\alpha + d_{\beta'} d_\alpha + d_\beta d_{\alpha'} + d_{\beta'} d_{\alpha'} + d_\beta \sum_{i=1}^{t} d_{\beta_i} + d_\beta \sum_{i=1}^{t} d_{\beta_i'} \\ &\quad + d_{\beta'} \sum_{i=1}^{t} d_{\beta_i} + d_{\beta'} \sum_{i=1}^{t} d_{\beta_i'} + d_\alpha \sum_{i=1}^{t} d_{\alpha_i} + d_\alpha \sum_{i=1}^{t} d_{\alpha_i'} + d_{\alpha'} \sum_{i=1}^{t} d_{\alpha_i} + d_{\alpha'} \sum_{i=1}^{t} d_{\alpha_i'} \\ &= 4(2t+2)^2 + 8((2t+2)(2t)). \end{aligned}$$

□

Theorem 3.3. *The F-index and Zagreb index of shadow graph of path is*

$$FT(D_2(P_t)) = 128(t-2) + 32,$$

$$M_1(D_2(P_t)) = 32(t-2) + 16,$$

$$M_2(D_2(P_t)) = 64(t-3)^2 + 64.$$

Proof. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of the path P_t and $\alpha_1', \alpha_2', \ldots, \alpha_t'$ be the newly added vertices corresponding to the vertices $\alpha_1, \alpha_2, \ldots, \alpha_t$ in order to obtain $D_2(P_t)$. Then

$$|V(D_2(P_t))| = 2t \quad \text{and} \quad |E(D_2(P_t))| = 4t - 4.$$

Furthermore, the degree of each vertex of $D_2(P_t)$ may be computed as

$$\begin{aligned} d_{\alpha_1} &= d_{\alpha_t} = 2, \\ d_{\alpha_1'} &= d_{\alpha_t'} = 2, \\ d_{\alpha_i} &= \sum_{i=2}^{t-1} 4, \\ d_{\alpha_i'} &= \sum_{i=2}^{t-1} 4. \end{aligned}$$

Hence we can calculate the forgotten index of $D_2(P_t)$ as

$$\begin{aligned}
FT(D_2(P_t)) &= d^3_{\alpha_1} + d^3_{\alpha_t} + d^3_{\alpha'_1} + d^3_{\alpha'_t} + \sum_{i=2}^{t-1} d^3_{\alpha_i} + \sum_{i=2}^{t-1} d^3_{\alpha'_i} \\
&= 128(t-2) + 32, \\
M_1(D_2(P_t)) &= d^2_{\alpha_1} + d^2_{\alpha_t} + d^2_{\alpha'_1} + d^2_{\alpha'_t} + \sum_{i=2}^{t-1} d^2_{\alpha_i} + \sum_{i=2}^{t-1} d^2_{\alpha'_i} \\
&= 32(t-2) + 16, \\
M_2(D_2(P_t)) &= d_{\alpha_1}d_{\alpha_2} + d_{\alpha_1}d_{\alpha'_2} + d_{\alpha'_1}d_{\alpha'_2} + d_{\alpha'_1}d_{\alpha_2} + d_{\alpha_{t-1}}d_{\alpha_t} + d_{\alpha_{t-1}}d_{\alpha'_t} + d_{\alpha'_{t-1}}d_{\alpha'_t} \\
&\quad + d_{\alpha'_{t-1}}d_{\alpha_t} + \sum_{i=2}^{t-2} d_{\alpha_i}d_{\alpha_{i+1}} + \sum_{i=2}^{t-2} d_{\alpha'_i}d_{\alpha'_{i+1}} + \sum_{i=2}^{t-2} d_{\alpha_i}d_{\alpha'_{i+1}} + \sum_{i=2}^{t-2} d_{\alpha'_i}d_{\alpha_{i+1}} \\
&= 64(t-3)^2 + 64.
\end{aligned}$$

□

Theorem 3.4. *The F-index and Zagreb index of shadow graph of cycle is*

$$FT(D_2(C_t)) = 128t,$$

$$M_1(D_2(C_t)) = 32t,$$

$$M_2(D_2(C_t)) = 64(t-1)^2 + 64.$$

Proof. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be the vertices of the first copy of the cycle C_t and $\beta_1, \beta_2, \ldots, \beta_t$ be the vertices of the second copy of the cycle C_t. We take

$$E(D_2(C_t)) = \{\alpha_i\alpha_{i+1}, \beta_i\beta_{i+1}, \alpha_i\beta_{i+1} : 1 \le i \le t-1\} \cup \{\alpha_i\beta_{i-1} : 2 \le i \le t\} \cup \{\alpha_t\alpha_1, \beta_t\beta_1, \alpha_t\beta_1, \alpha_1\beta_t\}.$$

Then

$$|V(D_2(C_t))| = 2t \quad \text{and} \quad |E(D_2(C_t))| = 4t.$$

In addition, the degree of each vertex of $D_2(C_t)$ may be calculated as

$$d_{\alpha_i} = d_{\beta_i} = \sum_{i=1}^{t} 4.$$

Thus we can compute the F-index and Zagreb of $D_2(C_t)$ as

$$\begin{aligned}
FT(D_2(C_t)) &= \sum_{i=1}^{t} d^3_{\alpha_i} + \sum_{i=1}^{t} d^3_{\beta_i} \\
&= 128t, \\
M_1(D_2(C_t)) &= \sum_{i=1}^{t} d^2_{\alpha_i} + \sum_{i=1}^{t} d^2_{\beta_i} \\
&= 32t,
\end{aligned}$$

$$\begin{aligned}
M_2(D_2(C_t)) &= \sum_{i=1}^{t-1} d_{\alpha_i}d_{\alpha_{i+1}} + \sum_{i=1}^{t-1} d_{\beta_i}d_{\beta_{i+1}} + \sum_{i=1}^{t-1} d_{\alpha_i}d_{\beta_{i+1}} + \sum_{i=2}^{t} d_{\beta_i}d_{\beta_{i-1}} + d_{\alpha_t}d_{\alpha_1} + d_{\beta_t}d_{\beta_1} + d_{\alpha_t}d_{\beta_1} + d_{\alpha_1}d_{\beta_t} \\
&= 64(t-1)^2 + 64.
\end{aligned}$$

□

4. Total graphs

In this section we find the total graphs of path and cycle to find the F-index and Zagreb index.

Theorem 4.1. *The F-index and Zagreb index of total graph of path is*

$$FT(T(P_t)) = 64(2t-5) + 70,$$

$$M_1(T(P_t)) = 16(2t-5) + 26,$$
$$M_2(T(P_t)) = 16(t-4)^2 + 48(t-3)^2 + 70.$$

Proof. Let $V(T(P_t)) = \{\alpha_i : 1 \le i \le t\} \cup \{\beta_i : 1 \le i \le t-1\}$ and

$$E(T(P_t)) = \{\beta_i\beta_{i+1} : 1 \le i \le t-2\} \cup \{\alpha_i\beta_{i-1} : 2 \le i \le t\} \cup \{\alpha_i\alpha_{i+1}, \alpha_i\beta_i : 1 \le i \le t-1\}.$$

Then
$$|V(T(P_t))| = 2t-1 \quad \text{and} \quad |E(T(P_t))| = 4t-5.$$

Furthermore, the degree of each vertex of $T(P_t)$ may be calculated as follows:

$$\begin{aligned} d_{\alpha_1} &= d_{\alpha_t} = 2, \\ d_{\alpha_i} &= \sum_{i=2}^{t-1} 4, \\ d_{\beta_1} &= d_{\beta_{t-1}} = 3, \\ d_{\beta_i} &= \sum_{i=2}^{t-2} 4. \end{aligned}$$

Hence we can determine the F-index and Zagreb index of $T(P_t)$ as

$$\begin{aligned} FT(T(P_t)) &= d_{\alpha_1} + d_{\alpha_t} + d_{\beta_1} + d_{\beta_{t-1}} + \sum_{i=2}^{t-1} d_{\alpha_i}^3 + \sum_{i=2}^{t-2} d_{\beta_i}^3 \\ &= 64(2t-5) + 70, \\ M_1(T(P_t)) &= d_{\alpha_1}^2 + d_{\alpha_t}^2 + d_{\beta_1}^2 + d_{\beta_{t-1}}^2 + \sum_{i=2}^{t-1} d_{\alpha_i}^2 + \sum_{i=2}^{t-2} d_{\beta_i}^2 \\ &= 16(2t-5) + 26, \\ M_2(T(P_t)) &= d_{\alpha_1}d_{\beta_1} + d_{\alpha_1}d_{\alpha_2} + d_{\beta_1}d_{\beta_2} + d_{\beta_1}d_{\alpha_2} + d_{\alpha_t}d_{\beta_t} + d_{\alpha_{t-1}}d_{\alpha_t} + d_{\beta_{t-1}}d_{\beta_t} + d_{\beta_t}d_{\alpha_{t-1}} \\ &\quad + \sum_{i=2}^{t-3} d_{\beta_i}d_{\beta_{i+1}} + \sum_{i=3}^{t-1} d_{\alpha_i}d_{\beta_{i-1}} + \sum_{i=2}^{t-2} d_{\alpha_i}d_{\alpha_{i+1}} + \sum_{i=2}^{t-2} d_{\alpha_i}d_{\beta_i} \\ &= 16(t-4)^2 + 48(t-3)^2 + 70. \end{aligned}$$

□

Theorem 4.2. *The F-index and Zagreb index of total graph of cycle is*

$$FT(T(C_t)) = 128t,$$

$$M_1(T(C_t)) = 32t,$$
$$M_2(T(C_t)) = 48(t-1)^2 + 16t^2 + 48.$$

Proof. Let $V(T(C_t)) = \{\alpha_i, \beta_i : 1 \le i \le t\}$ and

$$E(T(C_t)) = \{\alpha_i\alpha_{i+1}, \beta_i\beta_{i+1} : 1 \le i \le t-1\} \cup \{\alpha_i\beta_i : 1 \le i \le t\} \cup \{\alpha_i\beta_{i-1} : 2 \le i \le t\} \cup \{\alpha_t\alpha_1, \beta_t\beta_1, \alpha_1\beta_t\}.$$

Then

$$|V(T(C_t))| = 2t \quad \text{and} |E(T(C_t))| = 4t.$$

Furthermore, the degree of each vertex of $T(C_t)$ may be calculated as

$$d_{\alpha_i} = d_{\beta_i} = \sum_{i=1}^{t} 4.$$

Thus we can obtain the F-index and Zagreb index of $T(C_t)$ as

$$\begin{aligned} FT(T(C_t)) &= \sum_{i=1}^{t} d_{\alpha_i}^3 + \sum_{i=1}^{t} d_{\beta_i}^3 \\ &= 128t, \\ M_1(T(C_t)) &= \sum_{i=1}^{t} d_{\alpha_i}^2 + \sum_{i=1}^{t} d_{\beta_i}^2 \\ &= 32t, \\ M_2(T(C_t)) &= \sum_{i=1}^{t-1} d_{\alpha_i} d_{\alpha_{i+1}} + \sum_{i=1}^{t-1} d_{\beta_i} d_{\beta_{i+1}} + \sum_{i=1}^{t} d_{\alpha_i} d_{\beta_i} + \sum_{i=2}^{t} d_{\alpha_i} d_{\beta_{i-1}} + d_{\alpha_t} d_{\alpha_1} + d_{\beta_t} d_{\beta_1} + d_{\alpha_1} d_{\beta_t} \\ &= 48(t-1)^2 + 16t^2 + 48. \end{aligned}$$

□

5. Conclusion

Here we have computed the forgotten index and the Zagreb index of degree based topological indices for splitting, shadow and total graphs.

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