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Some new results on minimal neutrosophic compact space

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Abstract

The article focuses on several notions on minimal neutrosphic compact space. We introduce almost neutrosophic minimal compact space and nearly neutrosophic minimal compact space. We provide a suitable counter example in order to study the properties of almost neutrosophic minimal compact space. We investigate their basic properties and characterization theorems. We define countably neutrosophic m-compact space and investigate some of its basic properties and characterization theorems. We introduce the notion of neutrosophic m-Lindelöf space and investigate its basic properties and results. We further study the relationship of countably neutrosophic m-compact space with other known spaces.

Keywords: Neutrosophic set, neutrosophic topological space, minimal space, continuity

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1. Introduction

Human beings are facing real life problems due to uncertainty. In order to overcome this uncertainty, Zadeh [14] introduced the notion of fuzzy set which was inadequate to control uncertainty. Thereafter, Atanassov [3] revealed the notion of intuitionistic fuzzy set with the help of membership and non-membership values. Smarandache [4, 5] introduced and further investigated on the notion of neutrosophic set (NS, in short) to deal with real life problems related to uncertainty, imprecise, indeterminacy and inconsistent data. Salama and Alblowi [6] invented the neutrosophic topological spaces (NTS, in short). Makai et al. [7] referred minimal space and further investigated by Madok [9] in topological spaces. Alimohammady and Roohi [1] revealed minimal structure in fuzzy topological spaces. Alimohammady and Roohi [2] introduced compactness in fuzzy minimal spaces. Min [8] introduced the notion of intuitionistic fuzzy minimal spaces. Pal et al. [10, 11] talked on grill and minimal continuity in NTS.

- **Motivation**: Pal and Dhar [12] introduced compactness in neutrosophic minimal spaces (N_m-space, in short). However, they did not investigate on almost neutrosophic minimal compact, countably neutrosophic minimal compact, neutrosophic minimal Lindelöf space. This gap in the literature motivates us to study several properties of neutrosophic compact, countably neutrosophic compact, neutrosophic Lindelöfness in minimal spaces.
- **Methods used**: The article is basically theoretical in nature. We have defined almost neutrosophic minimal compact, countably neutrosophic minimal compact, neutrosophic minimal Lindelöf space. We have stated and

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established different theorems of these newly defined concepts. The cases for which the results fail have been supported by providing counter examples.

- **Advantages of the results**: The results obtained in the article have made the direction to study the compactness in more general spaces like almost neutrosophic minimal compact,countably neutrosophic minimal compact, neutrosophic minimal Lindelöf space. The results will also motivate to study different types of compactness in neutrosophic soft minimal space, multitset topological minimal space.

The article proceeds as bellow. The following part provides a quick overview of several well-known definitions and findings that are pertinent to the topic. In section 3, we investigate some basic properties, results and theorems of the notion of compactness in N_m-spaces. Section 4 indicates the conclusion of the work.

2. Preliminaries

Here we present some definitions and results which are relevant for investigation of this work.

Definition 2.1 (*cf.* [5])**.** An NS $\mathcal{N}$ in a whole set $\widetilde{U}$ is a set where each element consists with *truthness*, falseness and indeterminacy membership values appear from three independent functions, denoted by $f_{\mathcal{N}}$, $g_{\mathcal{N}}$, $h_{\mathcal{N}}$ in $[0, 1]$. Here $\mathcal{N}$ is given below:

$$\mathcal{N} = \left\{\langle \tilde{u}, f_{\mathcal{N}}, g_{\mathcal{N}}, h_{\mathcal{N}}\rangle : \tilde{u} \in \widetilde{U}\right\}$$

and each of $f_{\mathcal{N}}(\tilde{u})$, $g_{\mathcal{N}}(\tilde{u})$, $h_{\mathcal{N}}(\tilde{u})$ is a member of unit closed interval with the condition that sum of them lies between 0 to 3.

Definition 2.2 (*cf.* [5])**.** The collection $\widetilde{T}$ of NSs of a non-empty set $\widetilde{U}$ is termed as neutrosophic topology (NT, in short) on $\widetilde{U}$ if it obeys conditions:

(i) $0_{\mathcal{N}}$ and $1_{\mathcal{N}}$ are in $\widetilde{T}$.

(ii) The intersection of any two members of $\widetilde{T}$ is again in $\widetilde{T}$.

(iii) $\widetilde{T}$ is closed with respect to arbitrary union.

The order pair $(\widetilde{U}, \widetilde{T})$ is known as as NTS on $\widetilde{U}$. Members of $\widetilde{T}$ are referred *neutrosophic open sets* (NOS, in short) and their complements are *neutrosophic closed sets* (NCS, In short).

Remark 2.3. We denote 0_N and 1_N as the null and full NSs on a nonempty set $\widetilde{U}$ and these are defined as follows:

Definition 2.4 (*cf.* [6])**.** The NSs $0_{\mathcal{N}}$ and $1_{\mathcal{N}}$ in $\widetilde{\mathbb{U}}$ are represented as below:

(i) For any $x \in \widetilde{\mathbb{U}}$, $0_{\mathcal{N}} = (x, 0, 0, 0)$.

(ii) For any $x \in \widetilde{\mathbb{U}}$, $0_{\mathcal{N}} = (x, 0, 1, 0)$.

(iii) For any $x \in \widetilde{\mathbb{U}}$, $0_{\mathcal{N}} = (x, 0, 0, 1)$.

(iv) For any $x \in \widetilde{\mathbb{U}}$, $0_{\mathcal{N}} = (x, 0, 1, 1)$.

(v) For any $x \in \widetilde{\mathbb{U}}$, $1_{\mathcal{N}} = \{(x, 1, 1, 1)\}$.

(vi) For any $x \in \widetilde{\mathbb{U}}$, $1_{\mathcal{N}} = \{(x, 1, 1, 0)\}$.

(vii) For any $x \in \widetilde{\mathbb{U}}$, $1_{\mathcal{N}} = \{(x, 1, 0, 0)\}$.

(viii) For any $x \in \widetilde{\mathbb{U}}$, $1_{\mathcal{N}} = \{(x, 1, 0, 1)\}$.

Definition 2.5 (*cf.* [6])**.** For an NS $\mathcal{U} = \{(w, T_{\mathcal{U}}(w), I_{\mathcal{U}}(w), F_{\mathcal{U}}(w))\}$, the complement is

$$\mathcal{U}^c = \{(w, 1 - T_{\mathcal{U}}(w), 1 - I_{\mathcal{U}}(w), 1 - F_{\mathcal{U}}(w)\}, \text{ for any } w \in \mathcal{U}.$$

Definition 2.6 (*cf.* [6])**.** The *neutrosophic* interior of an NS U, denoted by $N_{int}(U)$, of an NTS $(\widetilde{U}, \widetilde{T})$ is the union of all NOSs E of $\widetilde{U}$ where $E \subseteq U$. The *neutrosophic* closure of an NS U, denoted by $N_{cl}(U)$, of an NTS $(\widetilde{U}, \widetilde{T})$ is the intersection of all NCSs F of $\widetilde{U}$ where $U \subseteq F$.

Remark 2.7 (*cf.* [6]). From the definition, it is evident that $N_{int}(U)$ is the largest NOS over $\widetilde{U}$ where $N_{int}(U) \subseteq U$ and $N_{cl}(U)$ is the smallest *NCS* on $\widetilde{U}$ where $U \subseteq N_{cl}(U)$.

Proposition 2.8 (*cf.* [6])**.** *For any NS B in* $(\widetilde{U}, \widetilde{T})$*, we have:*

(i) $(N_{int}(B^c)) = (N_{int}(B))^c$.

(ii) $(N_{cl}(B^c)) = (N_{cl}(B))^c$.

Definition 2.9 (*cf.* [10])**.** A family $\mathcal{M}$ of NSs of $\widetilde{U}$ (here $\mathcal{M} \subseteq \mathcal{P}(\widetilde{U})$) is called a *minimal structure* on $\widetilde{U}$ if both null and whole *NSs* are members of $\mathcal{M}$. We denote the N_m-space by $(\widetilde{U}, \mathcal{M})$. The elements of $\mathcal{M}$ are called *neutrosophic* $\mathfrak{m}$-open (N_m-open, in short) subsets of $\widetilde{U}$.

Definition 2.10 (*cf.* [10])**.** An NS F is called a *neutrosophic* $\mathfrak{m}$-closed (N_m-closed, in short) if its complement is an $N_m - open$ set.

Definition 2.11 (*cf.* [11])**.** The neutrosophic minimal interior and neutrosophic minimal closure of A, denoted by $N_{m-int}(A)$ and $N_{m-cl}(A)$, respectively, of an N_m-space $(\widetilde{U}, \mathcal{M})$ are defined respectively as

$$N_{m-int}(A) = \bigcup\{B : B \text{ is an } N_m\text{-open in } \widetilde{U} \text{ and } B \subseteq A\}.$$
$$N_{m-cl}(A) = \bigcap\{B : B \text{ is an } N_m\text{-closed in } \widetilde{U} \text{ and } A \subseteq B\}.$$

Definition 2.12 (*cf.* [11])**.** A function f from an N_m-space $(\widetilde{U}, \mathcal{M})$ to another N_m-space $(Y, \mathcal{N})$ is called N_m-continuous function if the inverse image of each N_m-open set in Y is N_m-open set in $\widetilde{U}$.

Definition 2.13 (*cf.* [13])**.** A NS $P = (x, T_P(x), I_P(x), F_P(x)) : x \in \widetilde{U}$ is called a neutrosophic point (NP, in short) of a non-empty set $\widetilde{U}$ iff for any element y in $\widetilde{U}$, we get, $T_P(y) = \alpha, I_P(y) = \beta, F_P(y) = \gamma$ for $y = x$ and $T_P(y) = 0, I_P(y) = 1, F_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1.0 \leq \beta < 1.0 \leq \gamma < 1$.

Definition 2.14 (*cf.* [13])**.** An NS A of an NTS $\widetilde{U}$ is called a neutrosophic neighbourhood or simply neighbourhood of an NP $x_{\alpha,\beta,\gamma}$ iff, $\exists$ an NS B in T where $x_{\alpha,\beta,\gamma} \in B \subseteq A$. A neighbourhood A of the NP $x_{\alpha,\beta,\gamma}$ is termed as neutrosophic open neighbourhood of $x_{\alpha,\beta,\gamma}$ if A is an NOS.

Definition 2.15 (*cf.* [11])**.** A function f from an N_m-space $(\widetilde{U}, \mathcal{M})$ to another N_m-space $(Y, \mathcal{N})$ is called N_m-open function if the image of each N_m-open set in $\widetilde{U}$ is N_m-open set in Y.

3. Main results

In this section, we present the main results.

Definition 3.1. Let $(\widetilde{U}, \mathcal{M})$ be an N_m-space. An NS A in $\widetilde{U}$ is called almost neutrosophic minimal compact (almost N_m-compact, in short) if for every N_m-open cover $\mathcal{A} = \{A_i \in \mathcal{NS}(\widetilde{U}) : i \in I\}$ of A, $\exists$ $J_0 = 1, 2, 3, \ldots \subseteq I$ where $A \subseteq \bigcup_{j \in J_0} N_m\text{-cl}(A_j)$. Here $\mathcal{NS}(\widetilde{U})$ means a set of all NSs on $\widetilde{U}$.

Theorem 3.2. *Let* $(\widetilde{U}, M)$ *be an* N_m*-space. If an NS A in* $\widetilde{U}$ *is* N_m*-compact, then it is also almost* N_m*-compact.*

Proof. Since the NS A in $\widetilde{U}$ is N_m-compact, so every N_m-open cover $\mathcal{B} = \{B_j : j \in I\}$ of A has a finite neutrosophic fuzzy subcover. This shows that A is also almost N_m-compact. □

Remark 3.3. The following example shows that the converse of Theorem 3.2. may not be true in general.

Example 3.4. Let $\widetilde{U}$ be a non-empty set and $n \in \mathbb{N} - \{1\}$. Let A_n be an NS defined as follows:
$A_n = \langle x, \mu A_n, \nu A_n, \gamma A_n \rangle$, $A_n = \langle x, \mu A_1, \nu A_1, \gamma A_1 \rangle$ by

$$\mu A_n(x) = \begin{cases} 0.7, & x = 0, \\ nx, & 0 < x \leq \frac{1}{n}, \\ 1, & \frac{1}{n} < x \leq 1. \end{cases}$$

$$\nu A_n(x) = \begin{cases} 0.2, & x = 0, \\ 1 - \frac{nx}{4}, & 0 < x \leq \frac{1}{n}, \\ 0.5, & \frac{1}{n} < x \leq 1. \end{cases}$$

$$\gamma A_n(x) = \begin{cases} 0.6, & x = 0, \\ 1 - \frac{nx}{4}, & 0 < x \leq \frac{1}{n}, \\ 0.4, & \frac{1}{n} < x \leq 1. \end{cases}$$

$$\mu A_1(x) = \begin{cases} 1, & x = 0 \\ 0.5, & \text{otherwise} \end{cases}$$

$$\nu A_1(x) = \begin{cases} 1, & x = 0 \\ 0.4, & \text{otherwise} \end{cases}$$

$$\gamma A_1(x) = \begin{cases} 0.5, & x = 0 \\ 0.6, & \text{otherwise.} \end{cases}$$

Consider an N_m-structure $\mathcal{M}$ on $\widetilde{U}$ as follows: $\mathcal{M} = \{A_n : n \in \mathbb{N}\} \cup \{0, 1\}$. Let $\mathcal{A} = \{A_n : n \in \mathbb{N}\}$ be an N_m-open cover of $\widetilde{U}$. Then, $\nexists$ a finite subcover of $\mathcal{A}$. Thus $\widetilde{U}$ is not N_m-compact. But $\widetilde{U}$ is almost N_m-space compact.

Theorem 3.5. *Consider N_m-continuous finction $f : (\widetilde{U}, M) \to (Y, N)$ between two N_m-spaces. If A is an almost N_m-compact, then $f(A)$ is also an almost N_m-compact set.*

Proof. Consider $\{B_i \in \mathcal{NS}(Y) : i \in J\}$ as N_m-open cover of $f(A)$ in Y. So $\{f^{-1}(B_i) : i \in J\}$ is a N_m-open cover of A in $\widetilde{U}$. By Definition 3.1., $\exists\ J_0 = \{1, 2, 3, \ldots, n\} \subseteq J$ such that $A \subseteq \bigcup_{j \in J_0} \mathcal{N}_{m-Cl}(f^{-1}(B_j))$. Hence it follows that $\bigcup_{j \in J_0} \mathcal{N}_{m-cl}(f^{-1}(B_j)) \subseteq \bigcup_{j \in J_0} f^{-1}(\mathcal{N}_{m-cl}(B_j)) = f^{-1}\left(\bigcup_{j \in J_0} \mathcal{N}_{m-cl}(f^{-1}(B_j))\right)$. Thus $f(A) \subseteq \bigcup_{j \in J_0} \mathcal{N}_{m-cl}(B_j)$. It completes the proof. □

Definition 3.6. Let $(\widetilde{U}, \mathcal{M})$ be an N_m-space. An NS A in $\widetilde{U}$ is termed nearly N_m-compact if for every N_m-open cover $\mathcal{A} = \{A_i \in \mathcal{I}(\widetilde{U}) : i \in I\}$ of A, $\exists\ J_0 = \{1, 2, 3, \ldots\} \subseteq I$ such that $A \subseteq \bigcup_{j \in J_0} \mathcal{N}_{m-int}\mathcal{I}(\mathcal{N}_{m-cl}(A_j))$.

Theorem 3.7. *Let $(\widetilde{U}, M)$ be an N_m-space. If an NS A in $\widetilde{U}$ is N_m-compact, then it is also nearly N_m-compact.*

Proof. For any N_m-open set A in $\widetilde{U}$, from Definition 3.5., it follows that $A \subseteq N_{m-int}(N_{m-cl}(A))$. Thus we get the result. □

Theorem 3.8. *Let $f : (\widetilde{U}, \mathcal{M}) \to (Y, \mathcal{N})$ be an N_m-continuous and N_m-open function on two N_m-spaces. If A is a nearly N_m-compact set, then $f(A)$ is also a nearly N_m-compact set.*

Proof. Consider $\{B_i \in \mathcal{NS}(Y) : i \in J\}$ as N_m-open cover of $f(A)$ in Y. So $\{f^{-1}(B_i) : i \in J\}$ is an N_m-open cover of A in $\widetilde{U}$. By nearly N_m-compactness, $\exists\ J_0 = \{1, 2, 3, \ldots\} \subseteq J$ such that $A \subseteq \bigcup_{j\in J_0} N_{m-int}(N_{m-cl}(f^{-1}(B_j)))$. It follows that

$$\begin{aligned} f(A) &\subseteq \bigcup_{j\in J_0} f(\mathcal{N}_{m-int}(\mathcal{N}_{m-cl}(f^{-1}(B_j)))) \\ &\subseteq 6\bigcup_{j\in J_0} \mathcal{N}_{m-int}(f(\mathcal{N}_{m-cl}(f^{-1}(B_j)))) \\ &\subseteq \bigcup_{j\in J_0} \mathcal{N}_{m-int}(f(f^{-1}(\mathcal{N}_{m-cl}B_j)))) \\ &\subseteq \bigcup_{j\in J_0} \mathcal{N}_{m-int}(\mathcal{N}_{m-cl}(B_j)). \end{aligned}$$

Hence $f(A)$ is a nearly N_m-compact set. □

Definition 3.9. An N_m-space $(\widetilde{U}, \mathcal{M})$ is named countably neutrosophic m-compact (countably N_m-compact, in short) if for every countable N_m-open sets $\{A_n : n \in \mathbb{N}\}$ in which $1_N = \bigcup_{n\in\mathbb{N}} A_n$, $\exists\ n_1, n_2, \ldots, n_k \in \mathbb{N}$ where $1_N = \bigcup_{i=1}^{k} A_{n_i}$.

Definition 3.10. An N_m-space $(\widetilde{U}, \mathcal{M})$ is named N_m-C_Π if there is a countable subfamily $\mathcal{B}$ of $\mathcal{M}$ such that any member of $\mathcal{M}$ can be expressed as the union of members in $\mathcal{B}$.

Theorem 3.11. *Suppose N_m-space $(\widetilde{U}, \mathcal{M})$ is neutrosophic m-C_Π. Then $\widetilde{U}$ is neutrosophic m-compact iff it is countably N_m-compact.*

Proof. Let us prove that $\widetilde{U}$ being N_m-compact is equivalent to being countably N_m-compact. Assume $\widetilde{U}$ is N_m-compact. Consider the collection $B_0 = \{B_i : j \in J, 1 \leq i \leq i_0\}$. By the m-compactness property, we can extract a finite subcover $B_1 \subseteq B_0$ that covers $\widetilde{U}$. As a consequence, any element of M can be represented as a union of elements from B_0. This establishes that $(\widetilde{U}, M)$ satisfies the countably N_m-compact condition.
For the converse, assume $\widetilde{U}$ is countably N_m-compact. Let $\mathcal{A} = \{A_j : j \in J\}$ be a collection of N_m-open sets where $1_N = \bigcup_{j\in J} A_j$. Since $\widetilde{U}$ satisfies the neutrosophic m-C_Π property, $\exists$ a countable subcollection $\mathcal{B}$ of $\mathcal{M}$ such that $\mathcal{A}^t = \bigcup_{i_0} \mathcal{B}_{i_0}$ (where i_0 may be infinite). Define $\mathcal{B}_0 = \{\mathcal{B}_i : j \in J, 1 \leq i \leq i_0\}$. One can verify that $\mathcal{B}_0$ constitutes a countable N_m-open cover of $\widetilde{U}$. Therefore, we can find a finite subcover $\mathcal{B}_1 \subseteq \mathcal{B}_0$ for $\widetilde{U}$. Since each element of $\mathcal{B}_1$ is contained within some A_j where $j \in J$, these corresponding A_j form the required finite subcover. □

Theorem 3.12. *An N_m-space $(\widetilde{U}, \mathcal{M})$ is countably N_m-compact iff every countable N_m-open cover has a finite partition.*

Proof. Suppose $A = \{A_j : j \in J\}$ is an N_m-cover. Since $\widetilde{U}$ is countably N_m-compact, so for every countable N_m-cover $\{A_n : n \in N\}$ in which $1_N = \bigcup_{n\in N} A_n$, $\exists n_1, n_2, ..., n_k \in N$ where $1_N = \bigcup_{i=1}^{k} A_{n_i}$. Hence $\bigcup_{i=1}^{k} A_{n_i}$ has a finite partition of $\widetilde{U}$. Since $\bigcup_{i=1}^{k} A_{n_i}$ is a subfamily of $\bigcup_{n\in N} A_n$, so every countable N_m-cover $\{A_n : n \in N\}$ has a finite partition.

Conversely, suppose that every countably N_m-cover $\{A_n : n \in N\}$ has a finite partition. Hence any countable N_m-cover $\{A_n : n \in N\}$ has a finite partition $\{F_{i,o} : i = 1, 2, ..., n\}$. Let A_i be an NS in correspondence of $F_{i,o}$. Now it is easy to see that $\{A_i : i = 1, 2, 3, ..., n\}$ is a finite subfamily of A which is also a neutrosophic cover of $\widetilde{U}$. □

From the Theorem 3.12, the Corollary 3.13 follows.

Corollary 3.13. *Suppose $(\widetilde{U}, \mathcal{M})$ is an N_m-space. Then $\widetilde{U}$ is not countable N_m-compact $\exists$ a countable N_m-open cover $\mathcal{A}$ of $\widetilde{U}$ and a point $x \in \widetilde{U}$ with $A_j(x)^c < 1\ \forall\ A_j \in \mathcal{A}$.*

Definition 3.14. An N_m-space $(\widetilde{U}, \mathcal{M})$ is termed N_m-Lindelöf if every N_m-open cover for $\widetilde{U}$ has a neutrosophic countable subcover.

Theorem 3.15. *An N_m-space $(\widetilde{U}, \mathcal{M})$ is N_m-Lindelöf iff $\bigcap_{j\in J} A_j \neq 1_N$ for any family $\{A_j : j \in J\}$ of N_m-closed sets in $\widetilde{U}$, where $\bigcap_{j\in K} A_j \neq 0_N$ for any countable subset K of J.*

Proof. Let $A_j : j \in J$ represent an N_m-open cover of $\widetilde{U}$. Consider the following: If for every countable subset $K \subseteq J$, we have $\bigcup_{j\in K} A_j \neq 1_N$, then it follows that $\bigcap_{j\in K} A_j^c \neq 0_N$. This leads to our assumption that $\bigcap_j A_j^c \neq 0_N$, which implies $\bigcap_j A_j \neq 1_N$. This produces a contradiction with our initial conditions. We can observe that in this scenario, any collection of N_m-closed sets exhibits the finite intersection property.We can conclude that $(\widetilde{U}, \mathcal{M})$ possesses both N_m-compactness and N_m-Lindelöf properties. □

As a consequence of Theorem 3.15 the Corollary 3.16 follows.

Corollary 3.16. *An N_m-space $(\widetilde{U}, \mathcal{M})$ is N_m-Lindelöf iff $\bigcap_{\mathcal{B}\in\mathcal{B}} m\text{-}cl(\mathcal{B}) \neq 0_N$ for every family $\mathcal{B}$ of NSs in $\widetilde{U}$, where the intersection of each countable subfamily of $\mathcal{B}$ is non-empty.*

Theorem 3.17. *Every N_m-C_{Π} space is an N_m-Lindelöf space.*

Proof. Consider an N_m-C_{Π} space $(\widetilde{U}, \mathcal{M})$ with an N_m-cover $\mathcal{A} = \{A_j : j \in J\}$. Given the properties of this space, we can find a countable subcollection $\mathcal{B} = \{\mathcal{B}_n : n \in \mathbb{N}\}$ within $\mathcal{A}$ such that each element A_j can be written as:

$$A_j = \bigcap_{i=1}^{j_0} B_i,$$

where j_0 could extend to infinity. Let us define $\mathcal{B}_0 = \{B_{ji} : \text{j} \in J, 1 \leq i \leq j_0\}$. We can verify that $\mathcal{B}$ is countable in nature. Furthermore, since every element of $\mathcal{B}_0$ is a subset of some A_j where j $\in J$, these corresponding A_j elements constitute a neutrosophic countable subcover of $\widetilde{U}$. □

Theorem 3.18. *Suppose $(\widetilde{U}, \mathcal{M})$ is an N_m-Lindelöf space and $\forall f : (\widetilde{U}, \mathcal{M}) \to (Y, \mathcal{N})$ is a surjective N_m - continuous function. Then Y is N_m-Lindelöf.*

Proof. Let $\{B_j : j \in J\}$ be a family of neutrosophic open cover of Y which satisfy $f(A) \leq \bigvee_j B_j$. Since f is continuous, so $A \leq f^{-1}(f(A)) \leq f^{-1}(\bigvee_{j\in J} B_j) = \bigvee_{j\in J} f^{-1}(B_j)$ and $\widetilde{U}$ is N_m-Lindelöf, $\exists j_1, j_2, \cdots \in N$ such that $A \leq \bigvee_{i=1}^{\infty} f^{-1}(B_{j_i})$, i.e., $A \leq f^{-1}(\bigvee_{i=1}^{\infty} B_{j_i})$. Consequently, $f(A) \leq f(f^{-1}(\bigvee_{i=1}^{\infty} B_{j_i})) \leq \bigvee_{i=1}^{\infty} B_{j_i}$, which completes the proof. □

Theorem 3.19. *An N_m-space is an N_m-Lindelöf space iff every N_m - open cover of $\widetilde{U}$ has countable ϵ-partition of $\widetilde{U}$ $\forall\, \epsilon$ with $0 < \epsilon < 1$.*

Proof. Let $(\widetilde{U}, \mathcal{M})$ denote an N_m-Lindelöf space and consider an N_m-cover $\mathcal{A} = \{A_j : j \in J\}$. By definition, $\mathcal{A}$ possesses a finite subcover, which we denote as $\mathcal{A}_0 = \{A_n : n \in \mathbb{N}\}$. For any real number ϵ where $0 < \epsilon < 1$, $\exists$ an ϵ-partition of $\widetilde{U}$ with respect to $\mathcal{A}_0$. Given that $\mathcal{A}_0$ is countable, this ϵ-partition of $\widetilde{U}$ must also be countable. Furthermore, since $\mathcal{A}_0 \subset \mathcal{A}$, this partition serves as an ϵ-partition with respect to $\mathcal{A}$ as well.
For the reverse direction, consider an N_m-cover $\mathcal{A}$ of $\widetilde{U}$ and fix $\epsilon \in (0, 1)$. Let $\Gamma_{i,0} : i \in I(c)$ represent a countable ϵ-partition of $\widetilde{U}$ with respect to $\mathcal{A}$, where each $\Gamma_{i,c}$ corresponds to an element $\mathcal{A}i, c$ in $\mathcal{A}$. Taking $\epsilon = \frac{1}{n}$ for integers $n \geq 2$, we can construct the collection $\mathcal{A}i, c : i \in I(c), \epsilon = \frac{1}{n}, n = 2, 3, \ldots$. This collection forms a countable subcover of the original cover $\mathcal{A}$. □

4. Conclusion

In this article, we have investigated the notions of some compactness in N_m-spaces. We have studied some properties of this newly defined compactness in N_m-spaces. Some characterization theorems have also been established in N_m-spaces. It is hoped that the notion of compactness in N_m-spaces can be applied in neutrosophic bi-topological spaces as well as in other areas of research.

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