Montes Taurus J. Pure Appl. Math. 8 (1), 63–76, 2026

Montes Taurus Journal of Pure and Applied Mathematics
https://www.mtjpamjournal.com
MTJPAM
ISSN: 2687-4814

Rough intersection graph of filters of a rough bi-Heyting algebra with application through hypergraph

Lourdusamy Packiammal Anto Freeda [a], Bashyam Praba [b]

[a] *Department of Mathematics, Sri Sivasubramaniya Nadar College of Engineering, Chennai, Tamil Nadu, India*
[b] *Department of Mathematics, Sri Sivasubramaniya Nadar College of Engineering, Chennai, Tamil Nadu, India*

Abstract

A rough bi-Heyting algebra is a mathematical structure extending classical logic with two implication operations on a rough semiring (T, Δ, ∇). It provides a robust framework for the algebraic study of uncertainty and approximation in knowledge systems. This paper investigates the rough intersection graph obtained through the filters of a rough bi-Heyting algebra, focusing on its structural properties and interrelations. It helps associate filters with the rough intersection graph's vertices and defines relations based on their non-empty non-identity intersections. The higher-order degree vertices in a rough intersection graph are identified based on the dense elements of a rough bi-Heyting algebra, which led to their characterization. The set of dense filters is denoted by $R - D_{\mathfrak{F}}(T)$. Later, the set of maximal filters being a proper subset of $R - D_{\mathfrak{F}}(T)$ is established. Furthermore, the results are extended to the domain of hypergraphs, where the complex relationships between filters are modeled through hyperedges. The significance of this study is explored to model the overlapping features shared by the different nodes in various contexts.

Keywords: Filter, dense filter, maximal filter, intersection graph, hypergraph

2020 MSC: 06D20, 05C65

1. Introduction

Rough Set Theory (RST) is a mathematical framework introduced by Pawlak [11] in the early 1980s for analyzing and modeling uncertainty and imprecision in data. It particularly deals with datasets where vagueness arises from insufficient or incomplete information. At its core, RST is based on approximating a set using a pair of sets, termed the lower approximation and the upper approximation, in an approximation space $I = (U, R)$. These approximations are derived from an indiscernibility relation R, which partitions the universe of discourse U into equivalence classes. The lower approximation represents the set of objects that are certainly members of the target set, while the upper approximation includes objects that could belong to the set. A rough set is defined through its lower and upper approximation and denoted by $RS(X)$ for $X \subseteq U$. The binary operations Δ and ∇ were introduced and defined specially for the rough sets in [17] by Praba and Mohan. Later, in [15], (T, Δ, ∇) was proved to be a semiring known as rough semiring. As the association of various algebraic structures with the graph has paved the way for many research groups, various graph characterizations were established on this rough semiring. To mention, see [16] for zero-divisor graph, see [9] for total complemented graph, and see [12] for rough identity-summand graph, etc. Many

†*Article ID:* MTJPAM-D-25-00033

Email addresses: antofreedalp@ssn.edu.in (Lourdusamy Packiammal Anto Freeda), prabab@ssn.edu.in (Bashyam Praba)
Received:31 January 2025, Accepted:13 November 2025, Published:13 March 2026
⋆*Corresponding Author:* Bashyam Praba

authors began defining various graphs, including intersection graphs over algebraic structures, in 1964. Bosak [3] was the one who initially proceeded in that way. An intersection graph derived from a semigroup S was defined and explored by Bosak [3]. He defined the proper subsemigroups of S as the graph's vertices and two separate vertices AB that are connected by an edge if and only if the intersection of the subsemigroups A and B is non-empty. Csákány and Pollak defined and examined the intersection graph of non-trivial proper subgroups of a finite group in 1969 (*cf.* [6]). Zelinka [19] carried on their research by examining the intersection graph of non-trivial subgroups of finite Abelian groups. In 2009, Chakrabarty et al. [5] introduced and examined the intersection graph of (nontrivial) ideals of a ring, demonstrating how numerous graph-theoretic properties-such as connectivity, completeness, cycles, etc reflect ring theoretic structure. They observed fascinating similarities between several algebraic aspects of rings and the graph-theoretic properties of the intersection graph. In [1], the work discusses various properties of the small intersection graph of filters of a bounded distributive lattice, such as being empty, complete, k-regular, triangle-free, or a tree, and provides classifications of lattices and results related to their properties. Whereas Atani et al. [2] introduces the concept of intersection graphs of co-ideals in semirings, denoted by $G(R)$. The authors of [2] discusses the conditions under which $G(R)$ is empty or connected and the relationship between the graph's properties and the algebraic properties of the semiring R. Several results are presented, including those relating to the diameter, clique number, and planarity of $G(R)$.

Because intersection graphs can represent interactions between substructures like ideals, subgroups, and filters, they have been extensively investigated for a variety of algebraic and topological structures. Classical intersection graphs, on the other hand, are limited in their ability to represent systems that require vagueness, incompleteness, or overlapping interactions since they are specified over simple or precisely defined sets. Elements may partially belong to different sets or show ambiguous class borders in a variety of real-world decision-making and data analysis challenges. The study of indiscernible or ambiguous connections is made possible by rough set theory, which offers a natural framework for approximation sets through lower and upper limits to resolve such ambiguity. Although intersection graphs and rough algebraic structures have been studied in great detail, the intersection graph of filters in a rough bi-Heyting environment has not yet been thoroughly investigated. Current frameworks either handle rough approximations or filters independently, failing to combine them into a single framework that can represent higher-order relations in a context of uncertainty. The approximate extensions are not covered in the literature currently available on intersecting graphs of algebraic structures. By extending the intersection graph of filters within the rough bi-Heyting algebraic framework, the goal of this study is to fill this gap by building and analyzing rough intersection graphs derived from filters. Examining their algebraic and topological characteristics, and showcasing their possible use in problems like multi-criteria decision-making and similarity detection.

Our proposed method in this study is one such direction of defining a graph called a rough intersection graph $R - IG_{\mathfrak{F}}(T)$ inspired by Ebrahimi et al. [1], a small intersection graph of filters of a lattice, and an intersection graph of co. ideals of a semiring (*cf.* [2]). An underlying structure taken in exploring the rough intersection graph is a rough bi-Heyting algebra (*cf.* [13]). A rough bi-Heyting algebra is an algebraic structure that differs from classical logic by the presence of a weaker notion of complements. In the study of structure theory, in general distributive lattices and in particular Boolean algebras, the notion of filters is essential. The concept of a filter and the graph corresponding to the filter were introduced in rough bi-Heyting algebra by Praba et al. [12, 14]. The rough identity-summand graph is a graph corresponding to the filter that helps in representing complex hierarchies. While exploring various filters of this rough bi-Heyting algebra, defining dense and maximal filters was motivated by [8, 10]. Also, there is a relation established between the dominant elements of T (*cf.* [16]), and the dense elements in this work. This study characterizes dense and maximal filters and explores their behavior on this rough bi-Heyting algebra. This is further taken to define the rough intersection graph of filters by considering the works of Ebrahimi et al. [1, 2]. This study is motivated by a need to capture the approximate and imprecise relationships between components of algebraic structures by extending the traditional concept of intersecting graphs into the rough-set framework. Through the use of the semiring structure on rough sets and the analysis of filters in rough bi-Heyting algebras, this paper develops a unified method for algebraic and graphical representation and analysis of uncertain connections. In addition to generalizing current graph-theoretic models, this extension strengthens the bond between soft computing, algebraic topology, and decision theory. Following from [1, 2], the rough intersection graph defined in this work analyzes the behavior of the vertices in the $R - IG_{(F)}(T)$, which plays the major role when the edge between two filters exists. So, the cardinality and the characterization of filters help to partition the vertices that behave similarly. Unlike $G(R)$ is a complete (or) regular graph, the rough intersection graph introduced in this study is a connected graph but not complete (or) regular. The adjacency between the vertices is highlighted, and it leads to characterizing the overlap between them. This overlap between the vertices captures its application in various fields. In [7], the research covers the mathematical and algorithmic properties of intersection graphs, their generalizations, and graph parameters, and Reff [18] establishes various matrix relationships between oriented hypergraphs and their intersection graphs, focusing on eigenvalue bounds and vertex/edge-switchings. Intersection graphs are widely studied in graph theory because they capture relationships between sets and have applications in various domains. Defining the rough intersection graph of filters of a rough bi-Hetyting algebra combines the algebraic properties with the graph theoretic notion, and represents this complex structure in the form of a hypergraph (*cf.* [4]). Also, converting this rough intersection graph into a hypergraph can be more applicable in finding similarity between the vertices. This leads to the overview of finding the similar behavior of elements in the overlapping vertices of each hyperedge in a hypergraph. The suggested framework illustrates how combining rough set theory with hypergraph structures offers a strong and adaptable tool for managing ambiguity and overlapping relationships in a variety of fields, including customer behavior analysis, cybersecurity threat classification, medical diagnosis, educational analytics, and industrial fault detection. In contrast to traditional pairwise or crisp relational approaches, the model goes above these restrictions by extracting a hypergraph from the rough intersection graph. Analysis of complex systems where elements naturally belong to numerous decision regions is made more accurate, interpretable, and adaptable by this enhanced representation. This methodology creates a unified algebraic–combinatorial framework that improves the relationship between theoretical rough set models and real-world data-driven applications.

This paper is divided into the following sections. In Subsection 1.1, some basic notions and formal definitions are provided. In Section 2, dense filters, maximal filters, and dominant elements of a rough bi-Heyting algebra are characterized. In Section 3, we define the rough intersection graph $R - IG_{\mathfrak{F}}(T)$ for the filters of a rough bi-Heyting algebra and discuss their nature. In Sections 4 and 5, the application of our proposed method and the conclusion are given.

1.1. Preliminaries

This subsection provides the formal definitions for the remaining sections.

Definition 1.1 (*cf.* [11])**.** The structure $I = (U, R)$ is an approximation space where U is a non-empty finite set of objects, called a universe, and R is an arbitrary equivalence relation on U. A partition induced by the relation R consists of an equivalence class denoted $[x]_R$, a subset of U. The lower and upper approximations defined for the subset X of U are given, respectively, as follows:

$$R_-(X) = \{x \in U \mid [x]_R \subseteq X\}$$

and

$$R^-(X) = \{x \in U \mid [x]_R \cap X \neq \emptyset\}.$$

If X is an arbitrary subset of U, then the rough set $RS(X)$ is an ordered pair $(R_-(X), R^-(X))$. The set of rough sets is denoted by T and defined by $T = \{RS(X) \mid X \subseteq U\}$.

Definition 1.2 (*cf.* [17])**.** Let $X, Y \subseteq U$. The Praba join of X and Y is denoted by $X\Delta Y$ and defined as $X\Delta Y = X \cup Y$ if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$, where $IW(X)$ is the number of equivalence classes in X.

If $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$, then the equivalence classes obtained by the union of X and Y are identified. The elements of that class belonging to Y are deleted, and the new set is named Y. Now, we obtain $X\Delta Y$. This process is repeated until

$$IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y).$$

Definition 1.3 (*cf.* [17])**.** If $X, Y \subseteq U$, then an element $x \in U$ is called pivot element, if $[x]_R \nsubseteq X \cap Y$, but $[x]_R \cap X \neq \emptyset$ and $[x]_R \cap Y \neq \emptyset$.

Definition 1.4 (*cf.* [17])**.** If $X, Y \subseteq U$, then the pivot elements of X and Y are called the pivot set of X and Y, denoted by $P_{X\cap Y}$.

Definition 1.5 (*cf.* [17])**.** Let $X, Y \subseteq U$. The Praba meet of X and Y is denoted as $X\nabla Y$ and defined by

$$X\nabla Y = \{x \in U \mid [x]_R \subseteq X \cap Y\} \cup P_{X\cap Y}.$$

Here each pivot element in $P_{X\cap Y}$ is the representative of that particular class.

Theorem 1.6 (*cf.* [17])**.** *For any two sets X, Y in U,*

- **(i)** *$RS(X\Delta Y)$ is the least upper bound of $RS(X)$ and $RS(Y)$.*
- **(ii)** *$RS(X\nabla Y)$ is the greatest lower bound of $RS(X)$ and $RS(Y)$.*

Theorem 1.7 (*cf.* [17])**.** *$T = \{RS(X) \mid X \subseteq U\}$ is a lattice called rough lattice.*

Theorem 1.8 (*cf.* [15])**.** *For any given approximation space $I = (U, R)$, (T, Δ, ∇) is a semiring called rough semiring.*

Remark 1.9 (*cf.* [13]). $RS(E - R^{-}(X)) = RS(\emptyset)$.

Definition 1.10 (*cf.* [8])**.** An element a of a lattice L is called a dense element if $a^* = \{0\}$ and the set of dense elements of L is defined by $D(L) = \{a \in L \mid a^* = 0\}$.

Definition 1.11 (*cf.* [16])**.** A subset X of U is said to be dominant if $X \cap X_i \neq \emptyset$ for $i = 1, 2, \ldots, n$.

Definition 1.12 (*cf.* [2])**.** Let R be a semiring. The intersection graph of co-ideals of R, denoted by $G(R)$, is the graph with all elements of $I^*(R) = \{I : I \neq \{1\}$ is a proper co-ideal of $R\}$ as vertices and two distinct vertices I and J are adjacent if and only if $I \cap J \neq \{1\}$.

Definition 1.13 (*cf.* [12])**.** For any $X \subseteq U$, define $F_X(T)$ by $F_X(T) = \{RS(Y) \mid Y = X\Delta V\Delta W, V \in P(E \setminus E_X), W \in P(B \setminus B_X)\}$, where E is the set of equivalence classes in U and E_X be the set of equivalence classes in X, B is the pivot set of representative elements of equivalence classes and B_X is the pivot set of representative elements in X of equivalence classes, whose cardinality is greater than 1.

Theorem 1.14 (*cf.* [14])**.** *For any $X \subseteq U$, the cardinality of $F_X(T)$ is $2^r 3^{m-(t+r)} 2^{n-(m+k)}$.*

Definition 1.15 (*cf.* [4])**.** A hypergraph H denoted by $H = (V; E = (e_i)_{i\in I})$ on a finite set V is a family $(e_i)_{i\in I}$, (I is a finite set of indexes) of subsets of V called hyperedges. Sometimes V is denoted by $V(H)$ and E by $E(H)$.

2. Dense and maximal filters

In this section, we characterize the dense and maximal filters and some of their basic properties on a rough bi-Heyting algebra.

Definition 2.1. Let $(T, \Delta, \nabla, ^*, ^+, \rightarrow, \leftarrow, RS(\emptyset), RS(U))$ be a rough bi-Heyting algebra. An element $RS(X) \in T$ is said to be dense if $RS(X)^* = RS(E - R^{-}(X)) = RS(\emptyset)$ and the set of dense elements of T is defined by

$$D^*(T) = \{RS(X) \in T \mid RS(X)^* = RS(E - R^{-}(X)) = RS(\emptyset)\}.$$

Theorem 2.2. *$D^*(T)$ is the set of dominant elements in T.*

Proof. Consider $RS(X) \in D^*(T)$ then $RS(X)^* = RS(\emptyset)$

$$\begin{aligned} &\Rightarrow \quad RS(E - R^{-}(X)) = RS(\emptyset) \quad \text{(from Remark 1.9)} \\ &\Rightarrow \quad E - R^{-}(X) = \emptyset \\ &\Rightarrow \quad E = R^{-}(X) \\ &\Rightarrow \quad X \text{ is dominant.} \end{aligned}$$

Conversely, if X is dominant then $X \cap X_i \neq \emptyset$ for $i = 1, 2, \ldots, n$

$$\begin{aligned} &\Rightarrow \quad R^-(X) = E \\ &\Rightarrow \quad E - R^-(X) = \emptyset \\ &\Rightarrow \quad RS(E - R^-(X)) = RS(\emptyset) \\ &\Rightarrow \quad RS(X)^* = RS(\emptyset) \\ &\Rightarrow \quad RS(X) \in D^*(T). \end{aligned}$$

□

Definition 2.3. For the set $B \subseteq U$, define $D^*(T) = \{RS(Z) \mid Z \in B\Delta(E \setminus R^-(B))\Delta P(R^-(B))\}$ where E is the set of equivalence classes in U and B is the pivot set consists of representative elements of the equivalence classes whose cardinality is greater than 1.

Theorem 2.4 (Characterization theorem for $D^*(T)$)**.** $D^*(T) = \{RS(Z) \mid Z \in B\Delta(E \setminus R^-(B))\Delta P(R^-(B))\}$.

Proof. Let $RS(X) \in D^*(T)$ then $RS(X)^* = RS(\emptyset)$

$$\begin{aligned} &\Leftrightarrow \quad R^-(X) = E \quad \text{(by Theorem 2.2)} \\ &\Leftrightarrow \quad X \in B\Delta(E \setminus R^-(B))\Delta P(R^-(B)) \\ &\Leftrightarrow \quad RS(X) \in \{RS(Z) \mid Z \in B\Delta(E \setminus R^-(B))\Delta P(R^-(B))\}. \end{aligned}$$

Therefore, $D^*(T) = \{RS(Z) \mid Z \in B\Delta(E \setminus R^-(B))\Delta P(R^-(B))\}$. □

Theorem 2.5. *$D^*(T)$ is a filter.*

Proof. Let $RS(X_1), RS(X_2) \in D^*(T)$, then $RS(X_1)^* = RS(X_2)^* = RS(\emptyset)$

$$\begin{aligned} &\Rightarrow \quad RS(X_1)^* \nabla RS(X_2)^* = RS(X_1 \nabla X_2)^* = RS(\emptyset) \\ &\Rightarrow \quad RS(X_1 \nabla X_2) \in D^*(T). \end{aligned}$$

$D^*(T)$ is closed under ∇. Now, suppose $RS(X_1) \in D^*(T)$ and $RS(X_2) \notin D^*(T)$

$$\begin{aligned} &\Rightarrow \quad RS(X_1)^* = RS(\emptyset) \text{ and } RS(X_2)^* \neq RS(\emptyset) \\ &\Rightarrow \quad RS(X_1 \Delta X_2) = RS(X_1) \\ &\Rightarrow \quad RS(X_1 \Delta X_2)^* = RS(X_1)^* = RS(\emptyset) \\ &\Rightarrow \quad RS(X_1 \Delta X_2) \in D^*(T). \end{aligned}$$

Therefore, $D^*(T)$ is a filter. □

Definition 2.6. A filter $F_X(T)$ is said to be a dense filter if $D^*(T) \subseteq F_X(T)$. The set of all dense filters of T is given by $D_{\mathfrak{F}}(T) = \{F_X(T) | X \in P(B \cup X_j)\}$, where $B = \{x_i \| X_i |> 1\}$ and $| X_j |= 1$ for $j \in \{m+1, \ldots, n\}$.

Remark 2.7. $D^*(T)$ is indeed a dense filter.

Theorem 2.8 (Characterization theorem for $D_{\mathfrak{F}}(T)$)**.** *$F_X(T)$ is a dense filter if $X \in P(B \cup X_j)$.*

Proof. Consider $X = \emptyset$, then

$$\begin{aligned} &F_{\{\emptyset\}}(T) = \{RS(Y) \mid Y \in P(E)\Delta P(B)\} \\ &\Rightarrow F_{\{\emptyset\}}(T) = T \\ &\Rightarrow D^*(T) \subseteq F_{\{\emptyset\}}(T). \end{aligned}$$

Therefore, $F_{\{\emptyset\}}(T)$ is a dense filter. Now consider $X = x_i$ where $| X_i | > 1$ for $i = \{1, 2, \ldots, m\}$ then $F_{\{x_i\}}(T) = \{RS(Y) | Y \in \{x_i\} \Delta P(E) \Delta P(B \setminus \{x_i\})\}$

$$\Rightarrow \quad F_{\{x_i\}}(T) = \{RS(Y) | Y \in (P(B) \setminus P(B \setminus \{x_i\}) \Delta P(E)\}$$
$$\Rightarrow \quad F_{\{x_i\}}(T) = R - upset(RS(\{x_i\})).$$

Let $RS(Y) \in D^*(T)$ then $RS(Y)^* = RS(\emptyset)$, for some $RS(Y) \in F_{\{x_i\}}(T)$ so that $RS(Y)^* = RS(\emptyset)$

$$\Rightarrow \quad D^*(T) \subseteq F_{\{x_i\}}(T)$$
$$\Rightarrow \quad F_{\{x_i\}}(T) \text{ is a dense filter.}$$

Similarly using the above arguments, the filter $F_{\{X_j\}}(T)$ is also a dense filter for $| X_j | = 1$ for $j \in \{m+1, m+2, \ldots, n\}$. □

Definition 2.9. Let $F_X(T)$ be a filter is said to be a maximal filter if

- **(i)** $F_X(T)$ is a proper filter.
- **(ii)** There is no proper filter $F_Y(T)$ such that $F_X(T) \subset F_Y(T)$.

Theorem 2.10 (Characterization theorem for maximal filter)**.** *$F_X(T)$ is a maximal filter if $X \in B \cup X_j$.*

Proof. When $X \in B$, $F_{\{x_i\}}(T) = \{RS(Y) \mid Y = \{x_i\} \Delta V \Delta W\}$ where $V \in P(E), W \in P(B \setminus \{x_i\})$

$$\Rightarrow \quad F_{\{x_i\}}(T) \neq T$$
$$\Rightarrow \quad F_{\{x_i\}}(T) \text{ is proper.}$$

Now to prove $F_{\{x_i\}}(T)$ is maximal, suppose there exists $F_Z(T) \in T \setminus F_{\{x_i\}}(T)$ and let $RS(S) \in F_Z(T)$

$$\Rightarrow RS(Y) \Delta RS(S) \in F_{\{x_i\}}(T).$$

If this is true for all $RS(Y)$ and $RS(S)$, then

$$F_Z(T) \subset F_{\{x_i\}}(T)$$
$$\Rightarrow F_{\{x_i\}}(T) \text{ is maximal.}$$

□

Theorem 2.11. *$F_X(T)$ is a maximal filter if $RS(X)$ is an atom.*

Proof. Assume $F_X(T)$ be a maximal filter

$$\Rightarrow \quad F_X(T) \text{ is proper}$$
$$\Rightarrow \quad F_X(T) \neq T$$
$$\Rightarrow \quad RS(X) \neq RS(\emptyset).$$

Also there exist no proper filter $F_Y(T)$ such that $F_X(T) \subset F_Y(T)$. Suppose there exist $F_Y(T)$ such that $F_X(T) \subset F_Y(T)$ then $Y = \emptyset$

$$\Rightarrow \quad RS(Y) \leq RS(X)$$
$$\Rightarrow \quad RS(X) \text{ is an atom.}$$

□

Remark 2.12. A maximal filter is one of the dense filters. The set of maximal filters is properly contained in the dense filters. The set of all maximal filters of T is denoted by $M_{\mathfrak{F}}(T) = \{F_X(T) | X \in B \cup X_j\}$, where $B = \{x_i \,||\, X_i | > 1\}$ and $| X_j | = 1$ for $j \in \{m + 1, \ldots, n\}$.

Theorem 2.13. *The set of all maximal filters of T is a subset of the set of all dense filters of T. Consequently, $M_{\mathfrak{F}}(T) \subset D_{\mathfrak{F}}(T)$.*

Proof. Let $F_X(T) \in M_{\mathfrak{F}}(T)$ then $F_X(T)$ is a proper filter for $X \in B \cup X_j$. By Theorem 2.10, $RS(X)$ is an atom.

$$\begin{aligned} &\Rightarrow \quad RS(X) \neq RS(\emptyset) \\ &\Rightarrow \quad F_X(T) \neq T. \end{aligned}$$

For all X such that $F_X(T) \neq T$ is given by $\mathfrak{F}(T) - F_{\emptyset}(T) = \{F_Z(T) \mid Z \in [P(B)\Delta P(E)] - \emptyset\}$. Since the maximal filter is one of the dense filters, the dense filters of $\mathfrak{F}(T) - F_{\emptyset}(T)$ is

$$\begin{aligned} \{F_Z(T) \mid Z \in P(B \cup (E \setminus R^{-}(B))) - \emptyset\} &= \{F_Z(T) \mid Z \in [P(B)\Delta P(E \setminus R^{-}(B))] - \emptyset\} \\ &= D_{\mathfrak{F}}(T) - F_{\emptyset}(T) \\ &\Rightarrow F_X(T) \in D_{\mathfrak{F}}(T) - F_{\emptyset}(T) \end{aligned}$$

for $X \in B \cup (E \setminus R^{-}(B))$

$$\Rightarrow M_{\mathfrak{F}}(T) \subset D_{\mathfrak{F}}(T).$$

□

3. Rough intersection graph of Filters and its hypergraph representation

Throughout this section, we discuss the nature and properties concerning the graph $R - IG_{\mathfrak{F}}(T)$.

Definition 3.1. The set of all filters of a rough bi-Heyting algebra is denoted by $\mathfrak{F}(T)$ and defined by $\mathfrak{F}(T) = \{F_X(T) \mid RS(X) \in T\}$.

Definition 3.2. A filter $F_X(T)$ is said to be non-trivial if $X \neq \emptyset$ and $X \neq U$.

Definition 3.3. Let $(T, \Delta, \nabla, ^{*}, ^{+}, \rightarrow, \leftarrow, RS(\emptyset), RS(U))$ be a rough bi-Heyting algebra. A rough intersection graph for the filters of T is denoted by $R - IG_{\mathfrak{F}}(T)$, whose vertex set is $V(R - IG_{\mathfrak{F}}(T)) = \{F_X(T) | F_X(T) \neq F_{\emptyset}(T)$ and $F_X(T) \neq F_U(T)\}$ and the edge between two distinct vertices $F_X(T)$ and $F_Y(T)$ exists iff $F_X(T) \cap F_Y(T) \neq \{RS(U)\}$.

Remark 3.4. From Definition 3.3, $| V(R - IG_{\mathfrak{F}}(T)) | = | \mathfrak{F}(T) | - 2$.

Theorem 3.5. *For any subset $X \subseteq U$ and the filter $F_X(T)$, the vertices non-adjacent with $F_X(T)$ in an rough intersection graph is*

$$\widetilde{F_X(T)} = \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\}.$$

Proof. The proof of the theorem is given in different cases for X:

Case 1. When $X = \{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}$ for $\{i_1, i_2, \ldots, i_r\} \in \{1, 2, \ldots, m\}$, the set of vertices non-adjacent with $\widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}}(T)}$ is

$$\widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}}(T)} = \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} = \{F_Z(T) \mid Z \in E - U\} = \emptyset.$$

$\therefore F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}}(T)$ is adjacent to all the vertices of a rough intersection graph.

Case 2. When $X = \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}$ for $\{s_1, s_2, \ldots, s_t\} \in \{1, 2, \ldots, m\}$, the set of vertices non-adjacent with $\widetilde{F_{\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)}$ is

$$\begin{aligned} \widetilde{F_{\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)} &= \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ &= \{F_Z(T) \mid Z \in [P(\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\})\Delta(E \setminus \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\})] - U\}. \end{aligned}$$

Case 3. When $X = \{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}$ for $\{j_1, j_2, \ldots, j_k\} \in \{m+1, m+2, \ldots, n\}$, the set of vertices non-adjacent with $\widetilde{F_{\{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)}$ is

$$\begin{aligned} \widetilde{F_{\{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)} &= \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ &= \{F_Z(T) \mid Z \in [P(\{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})\Delta(E \setminus \{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})] - U\}. \end{aligned}$$

Case 4. When $X = \{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}$ for $\{i_1, i_2, \ldots, i_r, s_1, s_2, \ldots, s_t\} \in \{1, 2, \ldots, m\}$, the set of vertices non-adjacent with $\widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)}$ is

$$\begin{aligned} \widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)} &= \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ &= \{F_Z(T) \mid Z \in [P(\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\})\Delta(E \setminus \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\})] - U\}. \end{aligned}$$

Case 5. When $X = \{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}$ for $\{i_1, i_2, \ldots, i_r\} \in \{1, 2, \ldots, m\}$ and $\{j_1, j_2, \ldots, j_k\} \in \{m+1, m+2, \ldots, n\}$, the set of vertices non-adjacent with $\widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)}$ is

$$\begin{aligned} \widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}\}}(T)} &= \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ &= \{F_Z(T) \mid Z \in [P(\{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})\Delta(E \setminus \{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})] - U\}. \end{aligned}$$

Case 6. When $X = \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}$ for $\{s_1, s_2, \ldots, s_t\} \in \{1, 2, \ldots, m\}$, $\{j_1, j_2, \ldots, j_k\} \in \{m+1, m+2, \ldots, n\}$, the set of vertices non-adjacent with $\widetilde{F_{\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)}$ is

$$\begin{aligned} &\widetilde{F_{\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)} \\ =\ &\{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ =\ &\{F_Z(T) \mid Z \in [P(\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})\Delta(E \setminus \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})] - U\}. \end{aligned}$$

Case 7. When $X = \{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}$ for $\{i_1, i_2, \ldots, i_r, s_1, s_2, \ldots, s_t\} \in \{1, 2, \ldots, m\}$ and $\{j_1, j_2, \ldots, j_k\} \in \{m+1, m+2, \ldots, n\}$, the set of vertices non-adjacent with $\widetilde{F_{\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)}$ is

$$\begin{aligned} &\widetilde{F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}, X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\}}(T)} \\ =\ &\{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\} \\ =\ &\{F_Z(T) \mid Z \in [P(\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})\Delta(E \setminus \{X_{s_1}, X_{s_2}, \ldots, X_{s_t}, X_{j_1}, X_{j_2}, \ldots, X_{j_k}\})] - U\}. \end{aligned}$$

□

Lemma 3.6. *For any $X \subseteq U$ and for the filter $F_X(T)$, the number of vertices non-adjacent with $F_X(T)$ in a rough intersection graph is*

$$\mid \widetilde{F_X(T)} \mid = 3^t 2^k - 1.$$

Proof. It is proved that $\widetilde{F_X(T)} = \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\}$.

If $F_Y(T) \in \widetilde{F_X(T)}$ and by Theorem 3.5, for $\{X_{s_1}, X_{s_2}, \ldots, X_{s_t}\} \subseteq X$ be the equivalence classes of cardinality greater than one will have 3 choices namely $\emptyset$ (or) x_i (or) X_i. Also for $\{X_{j_1}, X_{j_2}, \ldots, X_{j_k}\} \subseteq X$ be the equivalence classes of cardinality equal to one will have 2 choices either $\emptyset$ (or) X_j.

Also, for the given X, $F_X(T)$ is not adjacent to itself. Hence the total number of vertices non-adjacent to $F_X(T)$ will be $3^t 2^k - 1$. □

Lemma 3.7. *For the subset X of U, the degree of $F_X(T)$ in a rough intersection graph $R - IG_{\mathfrak{F}}(T)$ is*

$$deg(F_X(T)) = \mid V(R - IG_{\mathfrak{F}}(T)) \mid - \mid \widetilde{F_X(T)} \mid -1,$$

where $\widetilde{F_X(T)} = \{F_Z(T) \mid Z \in [P(B)\Delta P(E_X)\Delta(E \setminus E_X)] - U\}$.

Proof. The degree of vertices in a rough intersection graph $R - IG_{\mathfrak{F}}(T)$ will be given in different cases for various X.

Case 1. When $X = \{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}$ for $\{i_1, i_2, \ldots, i_r\} \in \{1, 2, \ldots, m\}$, the degree of $F_{\{x_{i_1}, x_{i_2}, \ldots, x_{i_r}\}}(T)$ is

$$deg(F_{\{x_{i_1},x_{i_2},\ldots,x_{i_r}\}}(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_{\{x_{i_1},x_{i_2},\ldots,x_{i_r}\}}}(T)\mid -1.$$

Using Theorem 3.5, $F_{\{x_{i_1},x_{i_2},\ldots,x_{i_r}\}}(T)$ is adjacent with all the vertices of $R-IG_{\mathfrak{F}}(T)$ and by Lemma 3.6,

$$deg(F_{\{x_{i_1},x_{i_2},\ldots,x_{i_r}\}}(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid -1.$$

Case 2. When $X=\{X_{s_1},X_{s_2},\ldots,X_{s_t}\}$ (or) $X=\{x_{i_1},x_{i_2},\ldots,x_{i_r},X_{s_1},X_{s_2},\ldots,X_{s_t}\}$ for $\{i_1,i_2,\ldots,i_r,\ s_1,s_2,\ldots,s_t\}\in\{1,2,\ldots,m\}$, the degree of $F_X(T)$ is

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1.$$

Then, using Theorem 3.5 and Lemma 3.6, we get

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1 =\mid V(R-IG_{\mathfrak{F}}(T))\mid -(3^t-1)-1.$$

Case 3. When $X=\{X_{j_1},X_{j_2},\ldots,X_{j_k}\}$ (or) $X=\{x_{i_1},x_{i_2},\ldots,x_{i_r},X_{j_1},X_{j_2},\ldots,X_{j_k}\}$ for $\{i_1,i_2,\ldots,i_r\}\in\{1,2,\ldots,m\}$ and $\{j_1,j_2,\ldots,j_k\}\in\{m+1,m+2,\ldots,n\}$, the degree of $F_X(T)$ is

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1.$$

Then, using Theorem 3.5 and Lemma 3.6, we have

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1 =\mid V(R-IG_{\mathfrak{F}}(T))\mid -(2^k-1)-1.$$

Case 4. When $X=\{X_{s_1},X_{s_2},\ldots,X_{s_t},X_{j_1},X_{j_2},\ldots,X_{j_k}\}$ (or) $X=\{x_{i_1},x_{i_2},\ldots,x_{i_r},X_{s_1},X_{s_2},\ldots,X_{s_t},X_{j_1},X_{j_2},\ldots,X_{j_k}\}$ for $\{i_1,i_2,\ldots,i_r,s_1,s_2,\ldots,s_t\}\in\{1,2,\ldots,m\}$ and $\{j_1,j_2,\ldots,j_k\}\in\{m+1,m+2,\ldots,n\}$, the degree of $F_X(T)$ is

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1.$$

Then, using Theorem 3.5 and Lemma 3.6, we get

$$deg(F_X(T)) =\mid V(R-IG_{\mathfrak{F}}(T))\mid - \mid \widetilde{F_X}(T)\mid -1 =\mid V(R-IG_{\mathfrak{F}}(T))\mid -(3^t2^k-1)-1.$$

□

Theorem 3.8. *In any rough intersection graph $R-IG_{\mathfrak{F}}(T)$, the elements common between any two vertices (say) $F_X(T)$ and $F_Y(T)$ are*

$$F_X(T)\cap F_Y(T)=F_{X\Delta Y}(T),$$

where $F_{X\Delta Y}(T)=\{RS(Z)\mid Z\in(X\Delta Y)\Delta P(E\setminus E_X\cup E_Y)\Delta P(B\setminus B_X\cup B_Y)\}$.

Proof. From Definition 1.13, $F_X(T)=\{RS(Y)\mid Y=X\Delta V\Delta W, V\in P(E\setminus E_X), W\in P(B\setminus B_X)\}$. To prove the theorem, let $RS(K)\in F_X(T)\cap F_Y(T)$, then

$$\begin{aligned}
\Leftrightarrow\quad & RS(K)\in\{RS(Z)\mid Z=X\Delta V_1\Delta W_1, V_1\in P(E\setminus E_X), W_1\in P(B\setminus B_X)\}\\
& \qquad\qquad \cap\{RS(Z)\mid Z=Y\Delta V_2\Delta W_2, V_2\in P(E\setminus E_Y), W_2\in P(B\setminus B_Y)\}\\
\Leftrightarrow\quad & RS(K)\in\{RS(Z)\mid Z\in(X\Delta Y)\Delta P(E\setminus E_X\cup E_Y)\Delta P(B\setminus B_X\cup B_Y)\}\\
\Leftrightarrow\quad & RS(K)\in F_{X\Delta Y}(T).
\end{aligned}$$

□

Corollary 3.9. *In any rough intersection graph $R-IG_{\mathfrak{F}}(T)$, the number of elements common to any two vertices (say) $F_X(T)$ and $F_Y(T)$ is*

$$\mid F_X(T)\cap F_X(T)\mid=\mid F_{X\Delta Y}(T)\mid=2^r3^{m-(t+r)}2^{n-(m+k)}.$$

Proof. The proof is straightforward. Theorem 1.14 identifies the cardinality of $F_{X\Delta Y}(T)$. □

Example 3.10. Consider $I = (U, R)$ be an approximation space, where the universal set $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and an arbitrary equivalence relation R induces the equivalence classes $X_1 = \{x_1, x_3\}$, $X_2 = \{x_2, x_4, x_6\}$ and $X_3 = \{x_5\}$ on U.
The rough sets obtained are

$$T = \{RS(\emptyset), RS(\{x_1\}), RS(\{x_2\}), RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(X_1 \cup X_2), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}.$$

Then, by the characterization given for the filter in Definition 1.13, the filters obtained for the subsets of U are $\mathfrak{F}(T) = \{F_X(T) | RS(X) \in T\}$ and are

- **(i)** $F_\emptyset(T) = T$
- **(ii)** $F_{\{x_1\}}(T) = \{RS(\{x_1\}), RS(X_1), RS(\{x_1\} \cup \{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(X_1 \cup X_3), RS(X_1 \cup X_2), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(iii)** $F_{\{x_2\}}(T) = \{RS(\{x_2\}), RS(X_2), RS(\{x_1\} \cup \{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(\{x_2\} \cup X_3), RS(X_2 \cup X_3), RS(X_1 \cup X_2), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(iv)** $F_{\{x_1,x_3\}}(T) = \{RS(X_1), RS(X_1 \cup \{x_2\}), RS(X_1 \cup X_3), RS(X_1 \cup X_2), RS(X_1 \cup \{x_2\} \cup X_3), RS(U)\}$
- **(v)** $F_{\{x_2,x_4,x_6\}}(T) = \{RS(X_2), RS(\{x_1\} \cup X_2), RS(X_2 \cup X_3), RS(X_1 \cup X_2), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(vi)** $F_{\{x_5\}}(T) = \{RS(X_3), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(vii)** $F_{\{x_1,x_2\}}(T) = \{RS(\{x_1\} \cup \{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(X_1 \cup X_2), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(viii)** $F_{\{x_1,x_2,x_4,x_6\}}(T) = \{RS(\{x_1\} \cup X_2), RS(X_1 \cup X_2), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(ix)** $F_{\{x_1,x_5\}}(T) = \{RS(\{x_1\} \cup X_3), RS(X_1 \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(x)** $F_{\{x_1,x_2,x_3\}}(T) = \{RS(X_1 \cup \{x_2\}), RS(X_1 \cup X_2), RS(X_1 \cup \{x_2\} \cup X_3), RS(U)\}$
- **(xi)** $F_{\{x_2,x_5\}}(T) = \{RS(\{x_2\} \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(xii)** $F_{\{x_1,x_2,x_3,x_4,x_6\}}(T) = \{RS(X_1 \cup X_2), RS(U)\}$
- **(xiii)** $F_{\{x_1,x_3,x_5\}}(T) = \{RS(X_1 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(U)\}$
- **(xiv)** $F_{\{x_2,x_4,x_5,x_6\}}(T) = \{RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(xv)** $F_{\{x_1,x_2,x_5\}}(T) = \{RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(xvi)** $F_{\{x_1,x_2,x_3,x_5\}}(T) = \{RS(X_1 \cup \{x_2\} \cup X_3), RS(U)\}$
- **(xvii)** $F_{\{x_1,x_2,x_4,x_5,x_6\}}(T) = \{RS(\{x_1\} \cup X_2 \cup X_3), RS(U)\}$
- **(xviii)** $F_{\{x_1,x_2,x_3,x_4,x_5,x_6\}}(T) = \{RS(U)\}$.

Here the dense and the maximal filters are $F_\emptyset(T), F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{X_3}(T), F_{\{x_1 \cup x_2\}}(T), F_{\{x_1 \cup X_3\}}(T), F_{\{x_2 \cup X_3\}}(T), F_{\{x_1 \cup x_2 \cup X_3\}}(T)$ and $F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{X_3}(T)$ respectively.
The vertices of a rough intersection graph are

$$V(R - IG_{\mathfrak{F}}(T)) = \{F_{\{x_1\}}(T), \ldots, F_{\{x_1 \cup X_2 \cup X_3\}}(T)\}.$$

The rough intersection graph $R - IG_{\mathfrak{F}}(T)$ obtained is given in Figure 1.

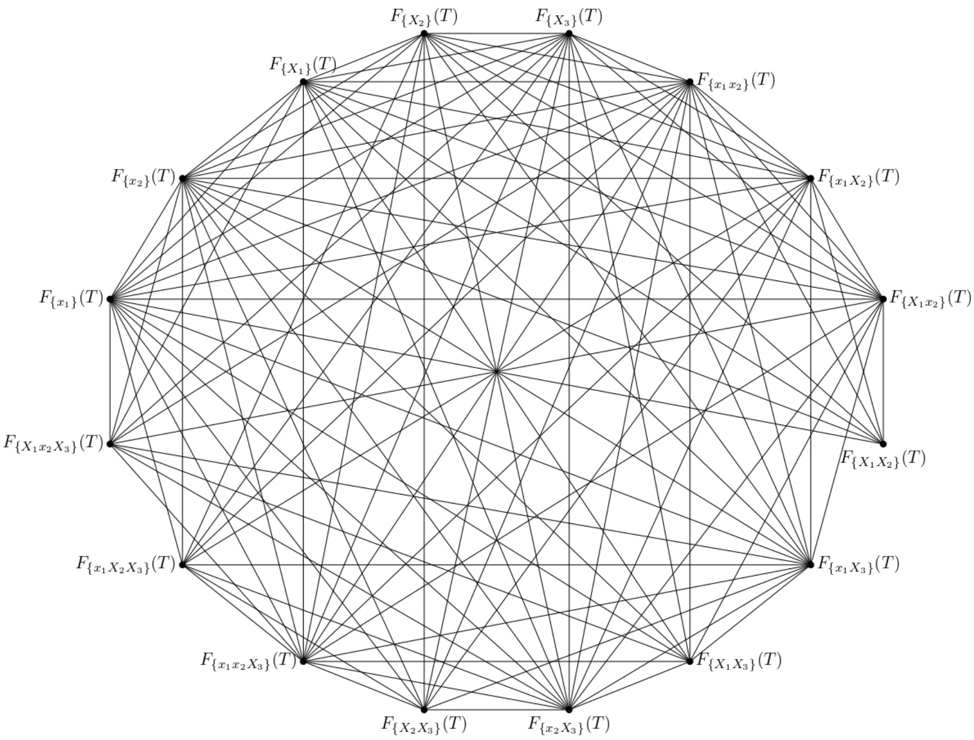

Figure 1. Rough intersection graph for the filters of T

The degree of vertices in Figure 1 is as follows:

(i) $deg(F_{\{x_1\}}(T)) = deg(F_{\{x_2\}}(T)) = deg(F_{\{x_1\cup x_2\}}(T)) = 15$

(ii) $deg(F_{X_3}(T)) = deg(F_{\{x_1\cup X_3\}}(T)) = deg(F_{\{x_2\cup X_3\}}(T)) = deg(F_{\{x_1\cup x_2\cup X_3\}}(T)) = 14$

(iii) $deg(F_{\{X_1\}}(T)) = deg(F_{\{X_2\}}(T)) = deg(F_{\{x_1\cup X_2\}}(T)) = deg(F_{\{X_1\cup x_2\}}(T)) = 13$

(iv) $deg(F_{\{X_1\cup X_3\}}(T)) = deg(F_{\{X_2\cup X_3\}}(T)) = deg(F_{\{x_1\cup X_2\cup X_3\}}(T)) = deg(F_{\{X_1\cup x_2\cup X_3\}}(T)) = 10$

(v) $deg(F_{\{X_1\cup X_2\}}(T)) = 7$.

Remark 3.11. For any rough intersection graph, its hypergraph representation involves the hyperedges, which are a subset of $V(R-IG_{\mathfrak{F}}(T)$. The vertices of each hyperedge share the overlapping elements, denoting the common feature shared by the vertices.

Example 3.12. For the rough intersection graph in Figure 1, its corresponding hypergraph is given in Figure 2. The hypergraph $H(R - IG_{\mathfrak{F}}(T)) = (V(R - IG_{\mathfrak{F}}(T), E)$. Here E is the subset of $V(R - IG_{\mathfrak{F}}(T)$ are the hyperedges. The hyperedges in Figure 2, is given by $E = \{e_1, e_2, \ldots, e_{13}\}$ where

(i) $e_1 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{x_1\cup x_2\}}(T)\}$

(ii) $e_2 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_3\}}(T), F_{\{x_1\cup x_2\}}(T), F_{\{x_1\cup X_3\}}(T), F_{\{x_2\cup X_3\}}(T), F_{\{x_1\cup x_2\cup X_3\}}(T)\}$

(iii) $e_3 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_2\}}(T), F_{\{x_1\cup x_2\}}(T), F_{\{x_1\cup X_2\}}(T)\}$

(iv) $e_4 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_2\}}(T), F_{\{X_3\}}(T), F_{\{x_1 \cup x_2\}}(T), F_{\{x_1 \cup X_2\}}(T), F_{\{x_1 \cup X_3\}}(T), F_{\{x_2 \cup X_3\}}(T), F_{\{X_2 \cup X_3\}}(T), F_{\{x_1 \cup x_2 \cup X_3\}}(T), F_{\{x_1 \cup X_2 \cup X_3\}}(T)\}$

(v) $e_5 = \{F_{\{x_1\}}(T), F_{\{X_3\}}(T), F_{\{x_1 \cup X_3\}}(T)\}$

(vi) $e_6 = \{F_{\{x_1\}}(T), F_{\{X_1\}}(T)\}$

(vii) $e_7 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_1\}}(T), F_{\{x_1 \cup x_2\}}(T), F_{\{X_1 \cup x_2\}}(T)\}$

(viii) $e_8 = \{F_{\{x_1\}}(T), F_{\{X_1\}}(T), F_{\{X_3\}}(T), F_{\{x_1 \cup X_3\}}(T), F_{\{X_1 \cup X_3\}}(T)\}$

(ix) $e_9 = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_1\}}(T), F_{\{X_2\}}(T), F_{\{x_1 \cup x_2\}}(T), F_{\{x_1 \cup X_2\}}(T), F_{\{X_1 \cup x_2\}}(T), F_{\{X_1 \cup X_2\}}(T)\}$

(x) $e_{10} = \{F_{\{x_1\}}(T), F_{\{x_2\}}(T), F_{\{X_1\}}(T), F_{\{X_3\}}(T), F_{\{x_1 \cup x_2\}}(T), F_{\{X_1 \cup x_2\}}(T), F_{\{x_1 \cup X_3\}}(T), F_{\{X_1 \cup X_3\}}(T), F_{\{x_2 \cup X_3\}}(T), F_{\{x_1 \cup x_2 \cup X_3\}}(T), F_{\{X_1 \cup x_2 \cup X_3\}}(T)\}$

(xi) $e_{11} = \{F_{\{x_2\}}(T), F_{\{X_2\}}(T)\}$

(xii) $e_{12} = \{F_{\{x_2\}}(T), F_{\{X_3\}}(T), F_{\{x_2 \cup X_3\}}(T)\}$

(xiii) $e_{13} = \{F_{\{x_2\}}(T), F_{\{X_2\}}(T), F_{\{X_3\}}(T), F_{\{x_2 \cup X_3\}}(T), F_{\{X_2 \cup X_3\}}(T)\}$.

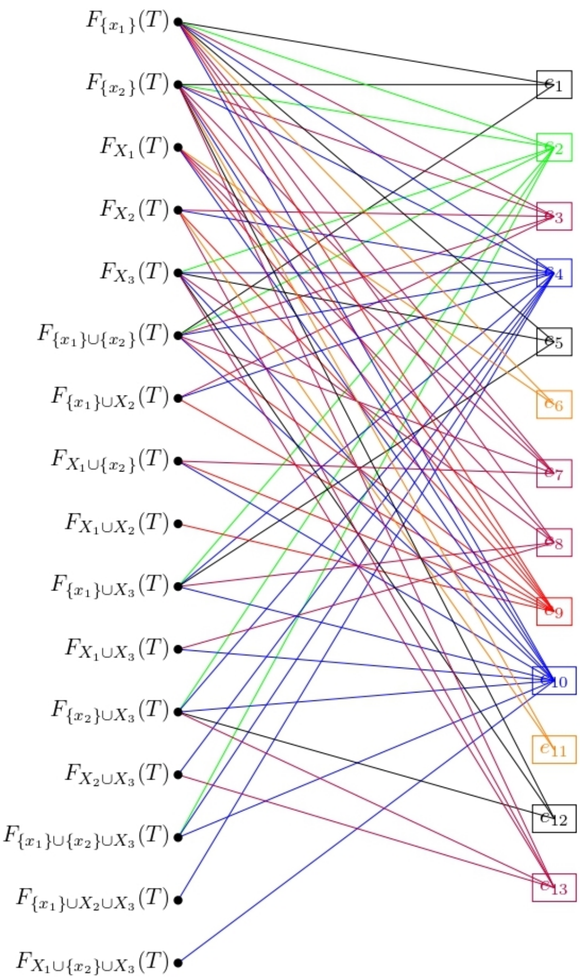

Figure 2. Hypergraph representation from the rough intersection graph of filters of T

Remark 3.13. For the vertices $F_X(T), F_Y(T) \in e$, where e is a hyperedge, the elements common between the vertices of a hyperedge e are $\{RS(Z)|Z \in [P(B_X \cup B_Y)\Delta P(E_X \cup E_Y)] - \emptyset\} = \{RS(Z)|Z \in [P(B_{X\cup Y})\Delta P(E_{X\cup Y})] - \emptyset\}$.

Remark 3.14. For the vertices $F_X(T), F_Y(T) \in e$, the number of elements common between the vertices of a hyperedge is $2^r 3^t 2^k - 1$.

4. Application

Consider the universal set U denotes the set of skills namely Communication skills (s_1), Problem-solving (s_2), Technical skills (s_3), Customer service (s_4), Teamwork and adaptability (s_5), Language proficiency (s_6), Work ethics (s_7), Professionalism (s_8), and Billing inquiries (s_9) required from the employee to be recruited for various departments in a company.

Let the departments be Customer service (D_1), Technical support (D_2), Sales (D_3), Quality assurance (D_4), Workforce management (D_5), Training and development (D_6), Human resource (D_7), and Data analytics and reporting (D_8).

The partition on U induces the equivalence classes

$$X_1 = \{s_1,\ s_6\}, X_2 = \{s_2, s_5\},\ X_3 = \{s_3, s_9\},\ X_4 = \{s_4, s_8\},\ X_5 = \{s_7\}.$$

If $X = \{\text{Work ethics}\}$, then $RS(X) = (X_5, X_5)$. This ensures that the employee is fit to work in the department D_4. Suppose that when $X = \{\text{Customer service, Technical skill}\}$, then $RS(X) = (\emptyset, X_3 \cup X_4)$ will not guarantee in which department the employee fits, or the employee cannot be placed in two departments simultaneously. So, the representation of a hypergraph from the rough intersection graph helps to cluster the employees with similar skills. The hyperedge represents such a cluster common to multiple rough sets. Each hyperedge will have common skills in terms of rough sets obtained from the vertices, which are collections of filters. This will lead to shared skill-based employee identification in the company's recruitment process. The methodology of this proposed study improves department-specific recruitment strategies by identifying clusters of essential skills.

5. Conclusion

The findings of this study highlight how rough intersection graphs and their hypergraph generalizations are powerful tools for visualizing and identifying the common features shared by the vertices. This helps to connect the rough set theory with graph-theoretic applications. The methodology of this work, which can even be further explored in various applications, is the key focus of our future work.

Acknowledgments

The authors thank the Management and the Principal of Sri Sivasubramaniya Nadar College of Engineering for providing the necessary facilities to carry out this work.

Author Contributions: The authors contributed equally to this work.

Conflict of Interest: The authors declare no conflict of interest.

Funding (Financial Disclosure): No funding.

References

[1] S. E. Atani, M. Khoramdel and M. Chenari, *The small intersection graph of filters of a bounded distributive lattice*, J. Mahani Math. Res. **12 (1)**, 311–326, 2023.
[2] S. E. Atani, S. D. Pishhesari, M. Khoramdel and Z. E. Sarvandi, *Intersection graphs of co-ideals of semirings*, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. **68 (1)**, 840–851, 2018.
[3] J. Bosák, *The graphs of semigroups*, In: Theory of Graphs and its Applications, Academic Press, New York, 119–125, 1964.
[4] A. Bretto, *Hypergraph theory: An introduction*, Springer, New York, 2013.

[5] I. Chakrabarty, S. Ghosh, T. K. Mukherjee and M. K. Sen, *Intersection graphs of ideals of rings*, Discrete Math. **309 (17)**, 5381–5392, 2009.
[6] B. Csákány and G. Pollak, *The graph of subgroups of a finite group*, Czechoslovak Math. J. **19 (2)**, 241–247, 1969.
[7] M. C. Golumbic, *Algorithmic aspects of intersection graphs and representation hypergraphs*, Graphs Combin. **4 (1)**, 307–321, 1988.
[8] A. P. Kumar, M. S. Rao and K. S. Babu, *Filters of distributive lattices generated by dense elements*, Palest. J. Math. **11 (2)**, 45–54, 2022.
[9] A. Manimaran, B. Praba and V. M. Chandrasekaran, *Total graph and complemented graph of a rough semiring*, Gazi Univ. J. Sci. **29 (2)**, 459–466, 2016.
[10] P. Nasehpour, *Pseudocomplementation and minimal prime ideals in semirings*, Algebra Universalis **79 (1)**, 1–15, 2018; Article ID: 11.
[11] Z. Pawlak, *Rough sets*, Int. J. Comput. Inform. Sci. **11**, 341–356, 1982.
[12] B. Praba and L. P. A. Freeda, *Generation of Gray codes through the rough identity–summand graph of filters of a rough bi–Heyting algebra*, Int. J. Appl. Math. Comput. Sci. **35 (2)**, 371–385, 2025.
[13] B. Praba and L. P. A. Freeda, *Rough bi-Heyting algebra and its applications in Heyting–Brouwer logic*, 2022; ArXiv:2208.11851v2.
[14] B. Praba and L. P. A. Freeda, *Characterization of rough identity-summand graph*, Asian-Eur. J. Math. **19 (2)**, 2026; Article ID: 2540023, https://doi.org/10.1142/ S1793557125400236.
[15] B. Praba, V. M. Chandrasekaran and A. Manimaran, *Semiring on rough set*, Indian J. Sci. Technol. **8 (2)**, 280–286, 2015.
[16] B. Praba, A. Manimaran and V. M. Chandrasekaran, *The zero divisor graph of a rough semiring*, Int. J. Pure Appl. Math. **98 (5)**, 33–37, 2015.
[17] B. Praba and R. Mohan, *Rough lattice*, Int. J. Fuzzy Math. Syst. **3 (2)**, 135–151, 2013.
[18] N. Reff, *Intersection graphs of oriented hypergraphs and their matrices*, Australas. J. Combin. **65 (1)**, 108–123, 2016.
[19] B. Zelinka, *Intersection graphs of finite abelian groups*, Czechoslovak Math. J. **25 (2)**, 171–174, 1975.

How to cite this article: L. P. A. Freeda and B. Praba, *Rough intersection graph of filters of a rough bi-Heyting algebra with application through hypergraph*, Montes Taurus J. Pure Appl. Math. **8 (1)**, 63–76, 2026; Article ID: MTJPAM-D-25-00033.